Cambridge International AS & A Level

MATHEMATICS (9709) P3

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS

Appendix A

Answers

1. 9709_s20_MS_31 Q: 2

(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2-3x)^{-2}$ or $\left(1-\frac{3}{2}x\right)^{-2}$	**M1**
	State correct first term $\frac{1}{4}$	**B1**
	Obtain the next two terms $\frac{3}{4}x+\frac{27}{16}x^2$	**A1 + A1**
		4
(b)	State answer $\lvert x\rvert<\frac{2}{3}$, or equivalent	**B1**
		1

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2. 9709_s20_MS_32 Q: 1

	Commence division and reach partial quotient $3x^2+kx$	**M1**
	Obtain quotient $3x^2+2x-1$	**A1**
	Obtain remainder $2x-5$	**A1**
		4

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3. 9709_s20_MS_33 Q: 1

	State or imply non-modular inequality $(2x-1)^2>3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x=-7$ and $x=-1$	**A1**
	State final answer $-7<x<-1$	**A1**
	Alternative method for question 1	
	Obtain critical value $x=-1$ from a graphical method, or by solving a linear equation or linear inequality	**B1**
	Obtain critical value $x=-7$ similarly	**B2**
	State final answer $-7<x<-1$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**
		4

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4. 9709_w20_MS_31 Q: 1

	Answer	Mark	Partial Marks
	Make a recognisable sketch graph of $y = 2\|x-3\|$ and the line $y = 2 - 5x$	B1	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	M1	Find point of intersection with $y = 2\|x-3\|$ or solve $2 - 5x$ with $2(x-3)$ or $-2(x-3)$
	Obtain $x = -\frac{4}{3}$	A1	
	State final answer $x < -\frac{4}{3}$	A1	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm\, 2(x-3)$	B1	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	M1	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\frac{4}{3}$	A1	
	State final answer $x < -\frac{4}{3}$	A1	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

5. 9709_w20_MS_31 Q: 9

	Answer	Mark	Partial Marks
(a)	State or imply the form $\frac{A}{1-x}+\frac{B}{2+3x}+\frac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\frac{A}{1-x}+\frac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
		5	
(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A = 1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B = 1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C = 6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	**A1 FT** + **A1 FT** + **A1 FT**	$(1+x+x^2)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$ $+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on *A*, *B*, *C*] $\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	**A1**	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D = -3, E = 4$ The FT is on *A*, *D*, *E*.
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6. 9709_w20_MS_32 Q: 2

	Answer	Mark	Partial Marks
(a)	State a correct unsimplified version of the x or x^2 or x^3 term	**M1**	For the given expression
	State correct first two terms $1 + 2x$	**A1**	
	Obtain the next two terms $-4x^2+\frac{40}{3}x^3$	**A1 + A1**	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
(b)	State answer $\lvert x\rvert<\frac{1}{6}$	**B1**	OE. Strict inequality
		1	

7. 9709_m19_MS_32 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $A+\frac{B}{2+x}+\frac{C}{3-2x}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -4$ and $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
(ii)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$ or $(3-2x)^{-1}$, or equivalent	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft +A1ft**	The ft is on B and C
	Add the value of A to the sum of the expansions	**M1**	
	Obtain final answer $2+\frac{7}{3}x+\frac{7}{18}x^2$	**A1**	
		5	

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8. 9709_s19_MS_31 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{2+x}+\frac{B}{3-x}+\frac{C}{(3-x)^2}$	**B1**	
	Use a correct method to obtain a constant	**M1**	
	Obtain one of $A = 2$, $B = 2$, $C = -7$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[Mark the form $\frac{A}{2+x}+\frac{Dx+E}{(3-x)^2}$, where $A = 2$, $D = -2$ and $E = -1$, B1M1A1A1A1.]
		5	
(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\frac{1}{2}x\right)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1** **A1** **A1**	**FT** on A, B and C $1-\frac{x}{2}+\frac{x^2}{4}$ $\frac{2}{3}\left(1+\frac{x}{3}+\frac{x^2}{9}\right)$ $-\frac{7}{9}\left(1+\frac{2x}{3}+\frac{3x^2}{9}\right)$
	Obtain final answer $\frac{8}{9}-\frac{43}{54}x+\frac{7}{108}x^2$	**A1**	
			For the A, D, E form of fractions give M1A1**ft**A1**ft** for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

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9. 9709_s19_MS_32 Q: 1

	Answer	Mark	Partial Marks
	State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$	**B1**	Symbolic binomial coefficients are not sufficient for the B marks
	State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$	**B1**	
	Multiply by $(3 - x)$ to give 2 terms in x^3, or their coefficients	**M1**	$\left(3\times\frac{10}{6}+1\right)$ Ignore errors in terms other than x^3 $3\times x^3\text{coeff} - x^2\text{coeff}$ and no other term in x^3
	Obtain answer 6	**A1**	Not $6x^3$
		4	

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10. 9709_s19_MS_33 Q: 9

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{3+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=-3$, $B=-1$, $C=2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Mark the form $\frac{A}{3+x}+\frac{Dx+E}{(1-x)^2}$, where $A=-3$, $D=1$ and $E=1$, B1M1A1A1A1 as above.
		5	
(ii)	Use a correct method to find the first two terms of the expansion of $(3+x)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(1-x)^{-1}$ or $(1-x)^{-2}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1**	**FT** on A
		A1	**FT** on B
		A1	**FT** on C
	Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2+\frac{190}{27}x^3$	**A1**	For the A, D, E form of fractions give M1A1ftA1ft for the expanded partial fractions, then, if $D\neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

11. 9709_w19_MS_31 Q: 2

	Answer	Mark	Partial Marks
	State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3) = \pm 4(x+1)$	**B1**	$12x^2 + 44x + 7 < 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Correct method seen, or implied by correct answers
	Obtain critical values $x = -\frac{7}{2}$ and $x = -\frac{1}{6}$	**A1**	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	**A1**	
	Alternative method for question 2		
	Obtain critical value $x = -\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = -\frac{1}{6}$ similarly	**B2**	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	**B1**	
		4	

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12. 9709_w19_MS_32 Q: 3

	Answer	Mark	Partial Marks
	Commence division and reach partial quotient $x^2 + kx$	**M1**	
	Obtain correct quotient $x^2 + 2x - 1$	**A1**	
	Set their linear remainder equal to 2x + 3 and solve for a or for b	**M1**	Remainder = $(a+3)x + (b-1)$
	Obtain answer a = – 1	**A1**	
	Obtain answer b = 4	**A1**	
	Alternative method for question 3		
	State $x^4 + 3x^3 + ax + b = (x^2 + x - 1)(x^2 + Ax + B) + 2x + 3$ and form and solve two equations in A and B	**M1**	e.g. $3 = 1 + A$ and $0 = -1 + A + B$
	Obtain $A = 2,\ B = -1$	**A1**	
	Form and solve equations for a or b	**M1**	e.g. $a = B - A + 2,\quad b = -B + 3$
	Obtain answer a = – 1	**A1**	
	Obtain answer b = 4	**A1**	
		5	
	Alternative method for question 3		
	Use remainder theorem with $x = \frac{-1 \pm \sqrt{5}}{2}$	**M1**	Allow for correct use of either root in exact or decimal form.
	Obtain $-\frac{a}{2} \pm \frac{a\sqrt{5}}{2} + b = \frac{9}{2} \mp \frac{\sqrt{5}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations for a or b	**M1**	
	Obtain answer a = – 1 from exact working	**A1**	
	Obtain answer b = 4 from exact working	**A1**	
		5	

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13. 9709_w19_MS_33 Q: 1

	Answer	Mark	Partial Marks
	State or imply non-modular inequality $(x+2)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x+2) = \pm(3x-1)$	B1	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = -\frac{3}{5}$ and $x = 5$	A1	
	State final answer $-\frac{3}{5} < x < 5$	A1	
	Alternative method for question 1		
	Obtain critical value $x = 5$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{3}{5}$ similarly	B2	
	State final answer $-\frac{3}{5} < x < 5$	B1	
		4	

14. 9709_w19_MS_33 Q: 2

	Answer	Mark	Partial Marks
	Substitute $x = -\frac{1}{2}$, equate result to zero and obtain a correct equation, e.g. $-\frac{6}{8} + \frac{1}{4}a - \frac{1}{2}b - 2 = 0$	B1	
	Substitute $x = -2$ and equate result to -24	*M1	
	Obtain a correct equation, e.g. $-48 + 4a - 2b - 2 = -24$	A1	
	Solve for a or for b	DM1	
	Obtain $a = 5$ and $b = -3$	A1	
		5	

15. 9709_m18_MS_32 Q: 2

	Answer	Mark
	State a correct unsimplified version of the x or x^2 or x^3 term	M1
	State correct first two terms $1 - x$	A1
	Obtain the next two terms $-\frac{3}{2}x^2 - \frac{7}{2}x^3$	A1 + A1
		4

16. 9709_s18_MS_31 Q: 4

	Answer	Mark
	EITHER: Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	**M1**
	Obtain quotient $x^2 + 3x + 2$	**A1**
	Either Set remainder identically equal to zero and solve for a or for b, *or* multiply given divisor and found quotient and obtain a or b	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
	OR: Assume an unknown factor $x^2 + Bx + C$ and obtain an equation in B and/or C	**M1**
	Obtain $B = 3$ and $A = 2$	**A1**
	Either Use equations to obtain a or b *or* multiply given divisor and found factor to obtain a or b	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
		5

17. 9709_s18_MS_31 Q: 9

	Answer	Mark
(i)	State or imply the form $A+\frac{B}{x-1}+\frac{C}{3x+2}$	**B1**
	State or obtain $A=4$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $B=3$, $C=-1$	**A1**
	Obtain the other value	**A1**
		5
(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$, or equivalent	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft + A1ft**
	Add the value of A to the sum of the expansions	**M1**
	Obtain final answer $\frac{1}{2}-\frac{9}{4}x-\frac{33}{8}x^2$	**A1**
		5

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18. 9709_s18_MS_32 Q: 9

	Answer	Mark	Partial Marks
(i)	Use a correct method to find a constant	**M1**	
	Obtain one of the values $A=-3$, $B=1$, $C=2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		4	
(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $\left(2+x^2\right)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients are not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1Ft + A1Ft**	The ft is on A, B and C. $-1\left(1+\frac{x}{3}+\frac{x^2}{9}+\frac{x^3}{27}...\right) \quad + \quad \frac{x+2}{2}\left(1-\frac{x^2}{2}...\right)$ $-1-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}+1-\frac{x^2}{2}+\frac{x}{2}-\frac{x^3}{4}$
	Multiply out their expansion, up to the terms in x^3, by $Bx+C$, where $BC \neq 0$	**M1**	
	Obtain **final** answer $\frac{1}{6}x-\frac{11}{18}x^2-\frac{31}{108}x^3$, or equivalent	**A1**	
		5	

19. 9709_s18_MS_33 Q: 1

	Answer	Mark
	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(4-3x)^{-\frac{1}{2}}$ or $\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$	**M1**
	State correct first term 2	**B1**
	Obtain the next two terms $\frac{3}{4}x+\frac{27}{64}x^2$	**A1 + A1**
	Total:	**4**

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20. 9709_w18_MS_31 Q: 1

	Answer	Mark	Partial Marks
	EITHER: State or imply non-modular inequality $2^2(2x-a)^2<(x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a)=\pm(x+3a)$	**B1**	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values $x=\frac{5}{3}a$ and $x=-\frac{1}{5}a$	**A1**	
	State final answer $-\frac{1}{5}a<x<\frac{5}{3}a$	**A1**	
	OR: Obtain critical value $x=\frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x=-\frac{1}{5}a$ similarly	**B2**	
	State final answer $-\frac{1}{5}a<x<\frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**	
		4	

21. 9709_w18_MS_32 Q: 1

	Answer	Mark	Partial Marks
	State or imply non-modular inequality $3^2(2x-1)^2 > (x+4)^2$, or corresponding quadratic equation, or pair of linear equations/inequalities $3(2x-1) = \pm(x+4)$	**B1**	$35x^2 - 44x - 7 = 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Allow for reasonable attempt at factorising e.g. $(5x - 7)(7x + 1)$
	Obtain critical values $x = \frac{7}{5}$ and $x = -\frac{1}{7}$	**A1**	Accept 1.4 and -0.143 or better for penultimate A mark
	State final answer $x > \frac{7}{5}$, $x < -\frac{1}{7}$	**A1**	'and' is A0, $\frac{7}{5} < x < -\frac{1}{7}$ is A0. Must be exact values. Must be strict inequalities in final answer
	Alternative		
	Obtain critical value $x = \frac{7}{5}$ from a graphical method	**B1**	or by inspection, or by solving a linear equation or an inequality
	Obtain critical value $x = -\frac{1}{7}$ similarly	**B2**	
	State final answer $x > \frac{7}{5}$ or $x < -\frac{1}{7}$ or equivalent	**B1**	[Do not condone $\geqslant$ for $>$, or $\leqslant$ for $<$.]
		4	

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22. 9709_w18_MS_32 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{1-2x} + \frac{B}{2-x} + \frac{C}{(2-x)^2}$	**B1**	
	Use a correct method for finding a constant M1 is available following a single slip in working from their form but no A marks (even if a constant is "correct")	**M1**	$7 = A + 2B$ $-15 = -4A - 5B - 2C$ $8 = 4A + 2B + C$
	Obtain one of $A = 1$, $B = 3$, $C = -2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	[Mark the form $\frac{A}{1-2x} + \frac{Dx+E}{(2-x)^2}$, where $A = 1$, $D = -3$ and $E = 4$, B1M1A1A1A1 as above.]		
		5	

	Answer	Mark	Partial Marks
(ii)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2-x)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2-x)^{-2}$ or $\left(1-\frac{1}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3ft**	The ft is on *A*, *B*, *C*. $1+2x+4x^2$ $\frac{3}{2}+\frac{3}{4}x+\frac{3}{8}x^2$ $-\frac{1}{2}-\frac{1}{2}x-\frac{3}{8}x^2$
	Obtain **final** answer $2+\dfrac{9}{4}x+4x^2$	**A1**	
	[For the *A*, *D*, *E* form of fractions give M1A2ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.]		[The ft is on *A*, *D*, *E*.]
		5	

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23. 9709_m17_MS_32 Q: 2

	Answer	Mark
	EITHER:	
	State or imply non-modular inequality $(x-4)^2 < (2(3x+1))^2$, or corresponding quadratic equation, or pair of linear equations $x-4 = \pm 2(3x+1)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x = -\frac{6}{5}$ and $x = \frac{2}{7}$	**A1**
	State final answer $x < -\frac{6}{5}$, $x > \frac{2}{7}$	**A1)**
	OR:	
	Obtain critical value $x = -\frac{6}{5}$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = \frac{2}{7}$ similarly	**B2**
	State final answer $x < -\frac{6}{5}$, $x > \frac{2}{7}$	**B1)**
	Total:	**4**

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24. 9709_m17_MS_32 Q: 9

	Answer	Mark
(i)	State or imply the form $\dfrac{A}{2+x}+\dfrac{Bx+C}{4+x^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = -2$, $B = 1$, $C = 4$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**

	Answer	Mark
(ii)	Use correct method to obtain the first two terms of the expansion of $(1+\frac{1}{2}x)^{-1}$, $(2+x)^{-1}$, $(1+\frac{1}{4}x^2)^{-1}$ or $(4+x^2)^{-1}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1√ + A1√**
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	**M1**
	Obtain final answer $\frac{3}{4}x-\frac{1}{2}x^2$	**A1**
	[Symbolic binomial coefficients, e.g. ${}_{-1}C_2$, are not sufficient for the first M1. The f.t. is on A, B, C.]	
	[In the case of an attempt to expand $x(6-x)(2+x)^{-1}(4+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	
	Total:	**5**

25. 9709_s17_MS_31 Q: 1

	Answer	Mark
	EITHER: State or imply non-modular inequality $(2x+1)^2 < (3(x-2))^2$, or corresponding quadratic equation, or pair of linear equations $(2x+1) = \pm 3(x-2)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for x	**M1**
	Obtain critical values $x = 1$ and $x = 7$	**A1**
	State final answer $x < 1$ and $x > 7$	**A1)**
	OR: Obtain critical value $x = 7$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = 1$ similarly	**B2**
	State final answer $x < 1$ and $x > 7$	**B1)**
	Total:	**4**

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26. 9709_s17_MS_31 Q: 2

	Answer	Mark
	EITHER: State a correct unsimplified version of the x or x^2 or x^3 term in the expansion of $(1+6x)^{-\frac{1}{3}}$	**(M1**
	State correct first two terms $1 - 2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ $\left(37\frac{1}{3}x^3\right)$ in final answer	**A1)**
	OR: Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x) = k(1+6x)^{-\frac{4}{3}}$	**(M1**
	Obtain correct first two terms $1 - 2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ in final answer	**A1)**
	Total:	**4**

27. 9709_s17_MS_32 Q: 2

	Answer	Mark
	EITHER: State or imply non-modular inequality $(x-3)^2 < (3x-4)^2$, or corresponding equation	(B1
	Make reasonable attempt at solving a three term quadratic	M1
	Obtain critical value $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	*OR*1: State the relevant critical inequality $3 - x < 3x - 4$, or corresponding equation	(B1
	Solve for x	M1
	Obtain critical value $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	*OR*2: Make recognizable sketches of $y = \lvert x-3 \rvert$ and $y = 3x - 4$ on a single diagram	(B1
	Find x-coordinate of the intersection	M1
	Obtain $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	Total:	4

28. 9709_s17_MS_32 Q: 8

	Answer	Mark
(i)	State or imply the form $\frac{A}{3x+2}+\frac{Bx+C}{x^2+5}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 1$, $C = -3$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**
(ii)	Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}$, $(1+\frac{3}{2}x)^{-1}$, $(5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient]	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction. The FT is on A, B, C. from part (i)	**A1FT + A1FT**
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	**M1**
	Obtain **final answer** $\frac{2}{5}-\frac{13}{10}x+\frac{237}{100}x^2$, or equivalent	**A1**
	Total:	**5**

29. 9709_s17_MS_33 Q: 2

	Answer	Mark
	EITHER: State a correct unsimplified version of the x or x^2 term in the expansion of $\left(1+\frac{2}{3}x\right)^{-3}$ or $(3+2x)^{-3}$ [Symbolic binomial coefficients, e.g. $\begin{pmatrix}-3\\2\end{pmatrix}$, are not sufficient for **M1**.]	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	OR: Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x)=k(3+2x)^{-4}$	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	Total:	**4**

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30. 9709_w17_MS_31 Q: 1

	Answer	Mark
	Commence division and reach a partial quotient x^2+kx	**M1**
	Obtain quotient x^2-2x+5	**A1**
	Obtain remainder $-12x+5$	**A1**
		3

31. 9709_w17_MS_32 Q: 8

	Answer	Mark
(i)	State or imply the form $\frac{A}{1-x}+\frac{B}{2x+3}+\frac{C}{(2x+3)^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 1$, $B = -2$, $C = 5$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
	[Mark the form $\frac{A}{1-x}+\frac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -4$, $E = -1$, **B1M1A1A1A1** as above.]	
(ii)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(1+\frac{2}{3}x)^{-1}$, $(2x+3)^{-1}$, $(1+\frac{2}{3}x)^{-2}$ or $(2x+3)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3 FT**
	Obtain final answer $\frac{8}{9}+\frac{19}{27}x+\frac{13}{9}x^2$, or equivalent	**A1**
		5

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32. 9709_m16_MS_32 Q: 4

		Answer	Mark	
(i)		Substitute $x=-\frac{1}{2}$ and equate to zero, or divide by $(2x+1)$ and equate constant remainder to zero	**M1**	
		Obtain $a = 3$	**A1**	[2]
(ii)	**(a)**	Commence division by $(2x + 1)$ reaching a partial quotient of $2x^2 + kx$	**M1**	
		Obtain factorisation $(2x+1)(2x^2-x+2)$	**A1**	[2]
		[The M1 is earned if inspection reaches an unknown factor $2x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 2$ and an equation in A and/or B.]		
	(b)	State or imply critical value $x=-\frac{1}{2}$	**B1**	
		Show that $2x^2-x+2$ is always positive, or that the gradient of $4x^3+3x+2$ is always positive	**B1***	
		Justify final answer $x>-\frac{1}{2}$	**B1(dep*)**	[3]

33. 9709_s16_MS_31 Q: 1

	Answer	Mark
(i)	*EITHER*: State or imply non-modular equation $(2(x-1))^2=(3x)^2$, or pair of linear equations $2(x-1)=\pm 3x$	B1
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations	M1
	Obtain answers $x=-2$ and $x=\frac{2}{5}$	A1
	OR: Obtain answer $x=-2$ by inspection or by solving a linear equation	(B1
	Obtain answer $x=\frac{2}{5}$ similarly	B2)
		[3]
(ii)	Use correct method for solving an equation of the form $5^x=a$ or $5^{x+1}=a$, where $a>0$	M1
	Obtain answer $x =- 0.569$ only	A1
		[2]

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34. 9709_s16_MS_31 Q: 8

	Answer	Mark
(i)	State or imply the form $\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$	B1
	Use a correct method to determine a constant	M1
	Obtain one of the values $A=1$, $B=3$, $C=12$	A1
	Obtain a second value	A1
	Obtain a third value	A1
		[5]
	[Mark the form $\frac{A}{x+1}+\frac{Dx+E}{(x-3)^2}$, where $A=1$, $D=3$, $E=3$, B1M1A1A1A1 as above.]	
(ii)	Use correct method to find the first two terms of the expansion of $(x+1)^{-1}, (x-3)^{-1}, (1-\frac{1}{3}x)^{-1}$, $(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√ + A1√ + A1√
	Obtain final answer $\frac{4}{3}-\frac{4}{9}x+\frac{4}{3}x^2$, or equivalent	A1
		[5]

35. 9709_s16_MS_32 Q: 2

Answer	Mark
State a correct un-simplified version of the x or x^2 or x^3 term	M1
State correct first two terms $1 + x$	A1
Obtain the next two terms $\frac{3}{2}x^2 + \frac{5}{2}x^3$	A1 A1 [4]
[Symbolic binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{2} \\ 3 \end{pmatrix}$ are not sufficient for the M mark.]	

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36. 9709_s16_MS_33 Q: 1

Answer	Mark
EITHER: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$	B1
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	M1
Obtain critical values $x = -5$ and $x = \frac{3}{5}$	A1
State final answer $-5 < x < \frac{3}{5}$	A1
OR: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality	(B1
Obtain critical value $x = \frac{3}{5}$ similarly	B2
State final answer $-5 < x < \frac{3}{5}$	B1)
[Do not condone ≤ for <.]	[4]

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37. 9709_s16_MS_33 Q: 10

Answer	Mark
(i) State or imply the form $\frac{A}{x+3} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$	B1
Use a correct method to determine a constant	M1
Obtain one of the values $A = -3$, $B = 1$, $C = 2$	A1
Obtain a second value	A1
Obtain the third value	A1
[Mark the form $\frac{A}{x+3} + \frac{Dx+E}{(x-1)^2}$, where $A = -3$, $D = 1$, $E = 1$, B1M1A1A1A1 as above.]	[5]
(ii) Use a correct method to find the first two terms of the expansion of $(x+3)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(x-1)^{-1}$, $(1-x)^{-1}$, $(x-1)^{-2}$, or $(1-x)^{-2}$	M1
Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction	A1√ + A1√ + A1√
Obtain final answer $\frac{10}{3}x + \frac{44}{9}x^2$, or equivalent	A1
	[5]

38. 9709_w16_MS_31 Q: 2

	Answer	Mark	
	State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$ State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$ Obtain sufficient terms of the product of $(2-x)$ and the expansion up to the term in x^2 Obtain **final** answer $2-7x+18x^2$ Do not ISW	**B1** **B1** **M1** **A1**	[4]

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39. 9709_w16_MS_33 Q: 4

	Answer	Mark	
(i)	Commence division by x^2-x+2 and reach a partial quotient $4x^2+kx$ Obtain quotient $4x^2+4x+a-4$ or $4x^2+4x+b/2$ Equate x or constant term to zero and solve for a or b Obtain $a = 1$ Obtain $b = -6$	**M1** **A1** **M1** **A1** **A1**	[5]
(ii)	Show that $x^2-x+2=0$ has no real roots Obtain roots $\frac{1}{2}$ and $-\frac{3}{2}$ from $4x^2+4x-3=0$	**B1** **B1**	[2]

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40. 9709_w16_MS_33 Q: 8

	Answer	Mark	
(i)	State or imply the form $\frac{A}{x+2}+\frac{Bx+C}{x^2+4}$ Use a correct method to determine a constant Obtain one of $A = 2$, $B = 1$, $C = -1$ Obtain a second value Obtain a third value	**B1** **M1** **A1** **A1** **A1**	[5]
(ii)	Use correct method to find the first two terms of the expansion of $(x+2)^{-1}$, $(1+\frac{1}{2}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$ Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction Multiply out fully by $Bx + C$, where $BC \neq 0$ Obtain final answer $\frac{3}{4}-\frac{1}{4}x+\frac{5}{16}x^2$, or equivalent [Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on A, B, C.] [In the case of an attempt to expand $(3x^2+x+6)(x+2)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	**M1** **A1**√ + **A1**√ **M1** **A1**	[5]

41. 9709_s15_MS_31 Q: 3

	Answer	Mark	
Either	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
	Obtain $1+4x^2$	A1	
	Obtain … $+12x^4$	A1	
	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
	Obtain $1+4x^2-4x^4$	A1	
	Combine expansions to obtain $k=16$ with no error seen	A1	
Or	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
	Obtain $1+4x^2$	A1	
	Obtain … $-4x^4$	A1	
	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
	Obtain $1+4x^2+12x^4$	A1	
	Combine expansions to obtain $k=16$ with no error seen	A1	[6]

42. 9709_s15_MS_32 Q: 8

	Answer	Mark	
(i)	State or imply the form $\dfrac{A}{3-2x}+\dfrac{Bx+C}{x^2+4}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A=3$, $B=-1$, $C=-2$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	**[5]**
(ii)	Use correct method to find the first two terms of the expansion of $(3-2x)^{-1}$, $(1-\frac{2}{3}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√+A1√	
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	M1	
	Obtain final answer $\frac{1}{2}+\frac{5}{12}x+\frac{41}{72}x^2$, or equivalent	A1	**[5]**

[Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient for the first M1. The f.t. is on A, B, C.]

[In the case of an attempt to expand $(5x^2+x+6)(3-2x)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

43. 9709_s15_MS_33 Q: 2

	Answer	Mark	
EITHER:	State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation	B1	
	Solve a 3-term quadratic, as in Q1.	M1	
	Obtain critical value $x = \frac{5}{3}$	A1	
	State final answer $x < \frac{5}{3}$ only	A1	
*OR*1:	State the relevant critical linear inequality $(2-x) > (2x-3)$, or corresponding equation	B1	
	Solve inequality or equation for x	M1	
	Obtain critical value $x = \frac{5}{3}$	A1	
	State final answer $x < \frac{5}{3}$ only	A1	
*OR*2:	Make recognisable sketches of $y = 2x - 3$ and $y = \lvert x-2 \rvert$ on a single diagram	B1	
	Find x-coordinate of the intersection	M1	
	Obtain $x = \frac{5}{3}$	A1	
	State final answer $x < \frac{5}{3}$ only	A1	**4**

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44. 9709_w15_MS_31 Q: 1

Answer	Mark	
EITHER: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$	**B1**	
Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**	
Obtain critical values -2 and $\frac{1}{4}$	**A1**	
State final answer $-2 < x < \frac{1}{4}$	**A1**	
OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**B1**	
Obtain critical value $x = \frac{1}{4}$ similarly	**B2**	
State final answer $-2 < x < \frac{1}{4}$	**B1**	[4]
[Do not condone $\leqslant$ for $<$]		

45. 9709_w15_MS_31 Q: 6

	Answer	Mark	
(i)	Substitute $x=-1$, equate to zero and simplify at least as far as $-8+a-b-1=0$	**B1**	
	Substitute $x=-\frac{1}{2}$ and equate the result to 1	**M1**	
	Obtain a correct equation in any form, e.g. $-1+\frac{1}{4}a-\frac{1}{2}b-1=1$	**A1**	
	Solve for a or for b	**M1**	
	Obtain $a = 6$ and $b = -3$	**A1**	[5]
(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2+kx$	**M1**	
	Obtain quadratic factor $8x^2-2x-1$	**A1**	
	Obtain factorisation $(x+1)(4x+1)(2x-1)$	**A1**	[3]
	[The M1 is earned if inspection reaches an unknown factor $8x^2+Bx+C$ and an equation in B and/or C, or an unknown factor Ax^2+Bx-1 and an equation in A and/or B.] [If linear factors are found by the factor theorem, give B1B1 for $(2x-1)$ and $(4x+1)$, and B1 for the complete factorisation.]		

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46. 9709_w15_MS_33 Q: 2

	Answer	Mark	
<u>Either</u>	State correct unsimplified x^2 or x^3 term	**M1**	
	Obtain $a=-9$	**A1**	
	Obtain $b=45$	**A1**	
<u>Or</u>	Use chain rule to differentiate twice to obtain form $k(1+9x)^{-\frac{5}{3}}$	**M1**	
	Obtain $f''(x)=-18(1+9x)^{-\frac{5}{3}}$ and hence $a=-9$	**A1**	
	Obtain $f'''(x)=270(1+9x)^{-\frac{8}{3}}$ and hence $b=45$	**A1**	[3]

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47. 9709_s20_MS_31 Q: 1

	Use law of the logarithm of a product or power	**M1**
	Obtain a correct linear inequality in any form, e.g. $\ln 2+(1-2x)\ln 3<x\ln 5$	**A1**
	Solve for x	**M1**
	Obtain $x>\frac{\ln 6}{\ln 45}$	**A1**
		4

48. 9709_s20_MS_32 Q: 2

	State or imply $2\ln y = \ln A + kx$	**B1**
	Substitute values of $\ln y$ and x, or equate gradient of line to k, and solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for $\ln A$	**M1**
	Obtain $A = 3.31$	**A1**
	Alternative method for question 2	
	Obtain two correct equations in y and x by substituting y- and x- values in the given equation	**B1**
	Solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for A	**M1**
	Obtain $A = 3.31$	**A1**
		5

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49. 9709_s20_MS_33 Q: 3

(a)	Remove logarithms correctly and state $1 + e^{-x} = e^{-2x}$, or equivalent	**B1**
	Show equation is $u^2 + u - 1 = 0$, where $u = e^x$, or equivalent	**B1**
		2
(b)	Solve a 3-term quadratic for u	**M1**
	Obtain root $\frac{1}{2}\left(-1+\sqrt{5}\right)$, or decimal in [0.61, 0.62]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer $x = -0.481$ only	**A1**
		4

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50. 9709_w20_MS_31 Q: 4

	Answer	Mark	Partial Marks
	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

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51. 9709_w20_MS_32 Q: 1

	Answer	Mark	Partial Marks
	State that $1+e^{-3x}=e^2$	**B1**	With no errors seen to that point
	Use correct method to solve an equation of the form $e^{-3x}=a$, where $a>0$, for x or equivalent	**M1**	$(e^{-3x}=6.389...)$ Evidence of method must be seen.
	Obtain answer $x=-0.618$ only	**A1**	Must be 3 decimal places
	Alternative method for question 1		
	State that $1+e^{-3x}=e^2$	**B1**	
	Rearrange to obtain an expression for e^x and solve an equation of the form $e^x=a$, where $a>0$, or equivalent	**M1**	$e^x=\sqrt[3]{\frac{1}{e^2-1}}$
	Obtain answer $x=-0.618$ only	**A1**	Must be 3 decimal places
		3	

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52. 9709_w20_MS_32 Q: 3

	Answer	Mark	Partial Marks
(a)	State or imply $y\log 2=\log 3-2x\log 3$	**B1**	Accept $y\ln 2=(1-2x)\ln 3$
	State that the graph of y against x has an equation which is linear in x and y, or is of the form $ay=bx+c$	**B1**	Correct equation. Need a clear statement/comparison with matching linear form.
	Clear indication that the gradient is $-\frac{2\ln 3}{\ln 2}$	**B1**	Must be exact. Any equivalent e.g. $-\frac{2\log_k 3}{\log_k 2}$, $\log_2\frac{1}{9}$
		3	
(b)	Substitute $y=3x$ in an equation involving logarithms and solve for x	**M1**	
	Obtain answer $x=\frac{\ln 3}{\ln 72}$	**A1**	Allow M1A1 for the correct answer following decimals
		2	

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53. 9709_m19_MS_32 Q: 1

	Answer	Mark	Partial Marks
(i)	Use law for the logarithm of a product or quotient	**M1**	
	Use $\log_{10}100=2$ or $10^2=100$	**M1**	
	Obtain $x^2-4x-100=0$, or equivalent	**A1**	
		3	
(ii)	Solve a 3-term quadratic equation	**M1**	
	Obtain answer 12.2 only	**A1**	
		2	

54. 9709_s19_MS_31 Q: 2

	Answer	Mark	Partial Marks
	Use law for the logarithm of a product, quotient or power	**M1**	Condone $\ln\frac{x}{x-1}$ for M1
	Obtain a correct equation free of logarithms	**A1**	e.g. $(2x-3)(x-1)=x^2$ or $x^2-5x+3=0$
	Solve a 3-term quadratic obtaining at least one root	**M1**	Must see working if using an incorrect quadratic $\left(\frac{5\pm\sqrt{13}}{2}\right)$
	Obtain answer $x = 4.30$ only	**A1**	Q asks for 2 dp. Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0
		4	

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55. 9709_s19_MS_32 Q: 2

	Answer	Mark	Partial Marks
	State or imply $u^2-u-12(=0)$, or equivalent in 3^x	**B1**	Need to be convinced they know $3^{2x}=\left(3^x\right)^2$
	Solve for u, or for 3^x, and obtain root 4	**B1**	
	Use a correct method to solve an equation of the form $3^x = a$ where a >0	**M1**	Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x\ln 3=\ln a, \quad x=\log_3 a$ If seen, accept solution of straight forward cases such as $3^x = 3$, $x = 1$ without working
	Obtain final answer $x = 1.26$ **only**	**A1**	The Q asks for 2 dp
		4	

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56. 9709_s19_MS_33 Q: 1

	Answer	Mark	Partial Marks
	Use law of the logarithm of a product or quotient	**M1**	
	Use law of the logarithm of power **twice**	**M1**	
	Obtain a correct linear equation in x, e.g. $(3-2x)\ln5=\ln4+x\ln7$	**A1**	
	Obtain answer $x = 0.666$	**A1**	
		4	

---

57. 9709_w19_MS_31 Q: 1

	Answer	Mark	Partial Marks
	State $1+e^{2y}=e^x$	**B1**	
	Make y the subject	**M1**	Rearrange to $e^{2y}=\ldots$ and use logs
	Obtain answer $y=\frac{1}{2}\ln\left(e^x-1\right)$	**A1**	OE
		3	

58. 9709_w19_MS_32 Q: 1

	Answer	Mark	Partial Marks
	Remove logarithms and state $4-3^x=e^{1.2}$, or equivalent	**B1**	Accept $4-3^x=3.32(01169.....)$ 3 s.f. or better
	Use correct method to solve an equation of the form $3^x=a$, where $a>0$.	**M1**	$(3^x=0.67988..)$ Complete method to $x=\ldots$ If using $\log_3$ the subscript can be implied
	Obtain answer $x=-0.351$ only	**A1**	CAO must be to 3 d.p.
		3	

---

59. 9709_w19_MS_33 Q: 3

	Answer	Mark	Partial Marks
	Reduce the equation to a horizontal equation in 3^{3x}, 3^{3x+1} or 27^x	**M1**	
	Simplify and reach $3(3^{3x})=5$, $3(27^x)=5$, or equivalent	**A1**	
	Use correct method for finding x from a positive value of 3^{3x}, 3^{3x+1} or 27^x	**M1**	
	Obtain answer $x=0.155$	**A1**	
		4	

---

60. 9709_m18_MS_32 Q: 4

	Answer	Mark
(i)	State or imply $n \ln y = \ln A + 3 \ln x$	**B1**
	State that the graph of $\ln y$ against $\ln x$ has an equation which is *linear* in $\ln y$ and $\ln x$, or has equation of the form $nY = \ln A + 3X$, where $Y = \ln y$ and $X = \ln x$, and is thus a straight line.	**B1**
		2
(ii)	Substitute x- and y-values in $n \ln y = \ln A + 3 \ln x$ or in the given equation and solve for one of the constants	**M1**
	Obtain a correct constant, e.g. $n = 1.70$	**A1**
	Solve for a second constant	**M1**
	Obtain the other constant, e.g. $A = 2.90$	**A1**
		4

---

61. 9709_s18_MS_31 Q: 1

	Answer	Mark
	Use law for the logarithm of a product, quotient or power	**M1**
	Obtain a correct equation free of logarithms, e.g. $4(x^4 - 4) = x^4$	**A1**
	Solve for x	**M1**
	Obtain answer x = 1.52 only	**A1**
		4

---

62. 9709_s18_MS_32 Q: 1

	Answer	Mark	Partial Marks
	EITHER: State or imply non-modular equation $3^2\left(2^x-1\right)^2=\left(2^x\right)^2$, or pair of equations $3\left(2^x-1\right)=\pm 2^x$	**M1**	$8\left(2^x\right)^2-18\left(2^x\right)+9=0$
	Obtain $2^x = \frac{3}{2}$ and $2^x = \frac{3}{4}$ or equivalent	**A1**	
	OR: Obtain $2^x = \frac{3}{2}$ by solving an equation	**B1**	
	Obtain $2^x = \frac{3}{4}$ by solving an equation	**B1**	
	Use correct method for solving an equation of the form $2^x = a$, where $a > 0$	**M1**	
	Obtain **final** answers x = 0.585 and x = – 0.415 only	**A1**	The question requires 3 s.f. Do not ISW if they go on to reject one value
		4	

---

63. 9709_s18_MS_33 Q: 2

	Answer	Mark
	State or imply $u^2 = u + 5$, or equivalent in 5^x	**B1**
	Solve for u, or 5^x	**M1**
	Obtain root $\frac{1}{2}(1+\sqrt{21})$, or decimal in [2.79, 2.80]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer x = 0.638 and no other answer	**A1**
	Total:	**5**

---

64. 9709_w18_MS_31 Q: 2

	Answer	Mark	Partial Marks
	Rearrange the equation in the form $ae^{2x} = b$ or $ae^{x} = be^{-x}$	**M1**	
	Obtain correct equation in either form with $a = 2$ and $b = 5$	**A1**	
	Use correct method to solve for x	**M1**	
	Obtain answer $x = 0.46$	**A1**	
		4	

---

65. 9709_w18_MS_32 Q: 4

	Answer	Mark	Partial Marks
	Substitute and obtain 3-term quadratic $3u^2 + 4u - 1 = 0$, or equivalent	**B1**	e.g. $3(e^x)^2 + 4e^x - 1 = 0$
	Solve a 3 term quadratic for u	**M1**	Must be an equation with real roots
	Obtain root $(\sqrt{7} - 2)/3$, or decimal in [0.21, 0.22]	**A1**	Or equivalent. Ignore second root (even if incorrect)
	Use correct method for finding x **from a positive value of** e^x	**M1**	Must see some indication of method: use of $x = \ln u$
	Obtain answer $x = -1.536$ only	**A1**	CAO. Must be 3 dp
		5	

---

66. 9709_m17_MS_32 Q: 1

	Answer	Mark
	Remove logarithm and obtain $1 + 2^x = e^2$	**B1**
	Use correct method to solve an equation of the form $2^x = a$, where $a > 0$	**M1**
	Obtain answer $x = 2.676$	**A1**
	Total:	**3**

---

67. 9709_s17_MS_32 Q: 1

	Answer	Mark
	Use law of the logarithm of a power or a quotient	**M1**
	Remove logarithms and obtain a correct equation in x. e.g. $x^2 + 1 = ex^2$	**A1**
	Obtain answer 0.763 and no other	**A1**
	Total:	**3**

---

68. 9709_s17_MS_33 Q: 3

	Answer	Mark
	Rearrange as $3u^2 + 4u - 4 = 0$, or $3e^{2x} + 4e^x - 4 = 0$, or equivalent	**B1**
	Solve a 3-term quadratic for e^x or for u	**M1**
	Obtain $e^x = \frac{2}{3}$ or $u = \frac{2}{3}$	**A1**
	Obtain answer $x = -0.405$ and no other	**A1**
	Total:	**4**

---

69. 9709_w17_MS_31 Q: 2

	Answer	Mark
	Plot the four points and draw straight line	**B1**
	State or imply that $\ln y = \ln C + x \ln a$	**B1**
	Carry out a completely correct method for finding $\ln C$ or $\ln a$	**M1**
	Obtain answer $C = 3.7$	**A1**
	Obtain answer $a = 1.5$	**A1**
		5

---

70. 9709_w17_MS_32 Q: 2

	Answer	Mark
	Use law for the logarithm of a power or a quotient on the given equation	**M1**
	Use $\log_2 8 = 3$ or $2^3 = 8$	**M1**
	Obtain $x^2 - 8x - 8 = 0$, or horizontal equivalent	**A1**
	Solve a 3-term quadratic equation	**M1**
	Obtain final answer $x = 8.90$ only	**A1**
		5

---

71. 9709_m16_MS_32 Q: 1

Answer	Mark	
Use law of the logarithm of a power, quotient or product	**M1**	
Remove logarithms and obtain a correct equation in x, e.g. $x^2+4=4x^2$	**A1**	
Obtain final answer $x=2/\sqrt{3}$, or exact equivalent	**A1**	[3]

---

72. 9709_s16_MS_32 Q: 1

Answer	Mark	
Use law of the logarithm of a product, power or quotient	**M1***	
Obtain a correct linear equation, e.g. $(3x-1)\ln 4=\ln 3+x\ln 5$	**A1**	
Solve a linear equation for x	**DM1***	
Obtain answer x = 0.975	**A1**	[4]

---

73. 9709_s16_MS_33 Q: 2

	Answer	Mark
(i)	State or imply $y\ln 3=(2-x)\ln 4$	**B1**
	State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent	**B1**
	State gradient is $-\frac{\ln 4}{\ln 3}$, or exact equivalent	**B1**
		[3]
(ii)	Substitute $y = 2x$ and solve for x, using a log law correctly at least once	**M1**
	Obtain answer $x=\ln 4/\ln 6$, or exact equivalent	**A1**
		[2]

---

74. 9709_w16_MS_31 Q: 1

	Answer	Mark	
	Solve for 3^x and obtain $3^x=\frac{18}{7}$	**B1**	
	Use correct method for solving an equation of the form $3^x=a$, where $a>0$	**M1**	
	Obtain answer x = 0.860 3 d.p. only	**A1**	[3]

---

75. 9709_w16_MS_33 Q: 1

	Answer	Mark	
	Use law of the logarithm of a quotient	**M1**	
	Remove logarithms and obtain a correct equation, e.g. $e^z=\frac{y+2}{y+1}$	**A1**	
	Obtain answer $y=\frac{2-e^z}{e^z-1}$, or equivalent	**A1**	[3]

76. 9709_s15_MS_31 Q: 1

Answer	**Mark**	
Use law for the logarithm of a power at least once	*M1	
Obtain correct linear equation, e.g. $5x\text{In}2 = (2x+1)\text{In}3$	A1	
Solve a linear equation for x	M1 dep *M	
Obtain $x = 0.866$	A1	[4]

---

77. 9709_s15_MS_32 Q: 2

Answer	**Mark**	
Use laws of indices correctly and solve for u	M1	
Obtain u in any correct form, e.g. $u = \dfrac{16}{16-1}$	A1	
Use correct method for solving an equation of the form $4^x = a$, where $a > 0$	M1	
Obtain answer $x = 0.0466$	A1	**[4]**

---

78. 9709_s15_MS_33 Q: 1

Answer	**Mark**	
Use law for the logarithm of a product, quotient or power	M1	
Obtain a correct equation free of logarithms, e.g. $\dfrac{x+4}{x^2} = 4$	A1	
Solve a 3-term quadratic obtaining at least one root	M1	
Obtain final answer $x = 1.13$ only	A1	**4**

---

79. 9709_w15_MS_31 Q: 2

Answer	**Mark**	
State or imply $1+u=u^2$	**B1**	
Solve for u	**M1**	
Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	**A1**	
Use correct method for finding x from a positive root	**M1**	
Obtain $x = 0.438$ and no other answer	**A1**	[5]

80. 9709_w15_MS_33 Q: 1

Answer	Mark	
Draw curve with increasing gradient existing for negative and positive values of x	**M1**	
Draw correct curve passing through the origin	**A1**	[2]

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81. 9709_s20_MS_31 Q: 3

	Use tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**
	Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan θ in any correct form	**A1**
	Reduce the equation to $3\tan^2\theta + 4\tan\theta - 1 = 0$, or equivalent	**A1**
	Solve a 3-term quadratic for tan θ	**M1**
	Obtain a correct answer, e.g. 12.1°	**A1**
	Obtain a second correct answer, e.g. 122.9°, and no others in the given interval	**A1**
		6

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82. 9709_s20_MS_32 Q: 5

(a)	State $R = \sqrt{7}$	**B1**
	Use trig formulae to find α	**M1**
	Obtain $\alpha = 57.688°$	**A1**
		3
(b)	Evaluate cos−1$\left(\frac{1}{\sqrt{7}}\right)$ to at least 3 d.p. (67.792°) (**FT** is on *their* R)	**B1 FT**
	Use correct method to find a value of θ in the interval	**M1**
	Obtain answer, e.g. 5.1°	**A1**
	Obtain second answer, e.g.117.3°, only	**A1**
		4

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83. 9709_s20_MS_33 Q: 5

	Use tan $2A$ formula to express RHS in terms of tan θ	**M1**
	Use tan $(A \pm B)$ formula to express LHS in terms of tan θ	**M1**
	Using tan 45° = 1, obtain a correct horizontal equation in any form	**A1**
	Reduce equation to $2\tan^2\theta + \tan\theta - 1 = 0$	**A1**
	Solve a 3-term quadratic and find a value of θ	**M1**
	Obtain answer $\theta = 26.6°$ and no other	**A1**
		6

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84. 9709_w20_MS_31 Q: 6

	Answer	Mark	Partial Marks
(a)	State $R = \sqrt{15}$	**B1**	
	Use trig formulae to find α	**M1**	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	**A1**	Must be 2 d.p. If radians 0.89 A0 MR
		3	
(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**	
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**	
		4	

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85. 9709_w20_MS_32 Q: 4

	Answer	Mark	Partial Marks
(a)	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\theta$	**M1**	e.g. $\frac{\tan\theta + \tan 60°}{1 - \tan\theta\tan 60°} = \frac{2}{\tan\theta}$
	Use $\tan 60° = \sqrt{3}$ and obtain a correct horizontal equation in any form	**A1**	e.g. $\tan\theta\left(\tan\theta + \sqrt{3}\right) = 2\left(1 - \sqrt{3}\tan\theta\right)$
	Reduce to $\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0$ correctly	**A1**	AG
		3	
(b)	Solve the given quadratic to obtain a value for θ	**M1**	$\left(\tan\theta = \frac{-3\sqrt{3} \pm \sqrt{35}}{2} = 0.3599, -5.556\right)$
	Obtain one correct answer e.g. $\theta = 19.8°$	**A1**	Accept 1d.p. or better. If over-specified must be correct. 19.797...., 100.2029...
	Obtain second correct answer $\theta = 100.2°$ and no others in the given interval	**A1**	Ignore answers outside the given interval.
		3	

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86. 9709_m19_MS_32 Q: 3

	Answer	Mark	Partial Marks
(i)	Use trig formulae and obtain an equation in sin θ and cos θ	**M1**	
	Obtain a correct equation in any form	**A1**	
	Substitute exact trig ratios and obtain an expression for tan θ	**M1**	
	Obtain answer $\tan\theta = \frac{2\sqrt{2}-1}{1-\sqrt{6}}$, or equivalent	**A1**	
		4	
(ii)	State answer, e.g. $\theta = 128.4°$	**B1**	
	State second answer, e.g. $\theta = 308.4°$	**B1 ft**	
		2	

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87. 9709_s19_MS_31 Q: 4

	Answer	Mark	Partial Marks
	Use correct trig formula and obtain an equation in tan θ	**M1**	Allow with $45°$ e.g. $\frac{1}{\tan\theta} - \frac{1}{\frac{\tan\theta+\tan 45°}{1-\tan\theta\tan 45°}} = 3$
	Obtain a correct horizontal equation in any form	**A1**	e.g. $1+\tan\theta-\tan\theta(1-\tan\theta) = 3\tan\theta(1+\tan\theta)$
	Reduce to $2\tan^2\theta + 3\tan\theta - 1 = 0$	**A1**	or 3-term equivalent
	Solve 3-term quadratic and find a value of θ	**M1**	Must see working if using an incorrect quadratic
	Obtain answer 15.7°	**A1**	One correct solution (degrees to at least 3 sf)
	Obtain answer 119.(3)°	**A1**	Second correct solution and no others in range (degrees to at least 3 sf) Mark 0.274, 2.082 as MR: A0A1
		6	

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88. 9709_s19_MS_31 Q: 6

	Answer	Mark	Partial Marks
(i)	State correct expansion of $\sin(2x+x)$	**B1**	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of sin x	**M1**	
	Obtain a correct expression in any form	**A1**	e.g. $2\sin x(1-\sin^2 x) + \sin x(1-2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	**A1**	Accept $=$ for $\equiv$
		4	
(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$	**B1 B1**	One mark for each term correct
	Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	**M1**	$\left(-\frac{3}{8} - \frac{1}{12} + \frac{3}{4} - \frac{1}{12} = -\frac{11}{24} + \frac{2}{3}\right)$
	Obtain answer $\frac{5}{24}$	**A1**	Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is 0/4
		4	

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89. 9709_s19_MS_32 Q: 3

	Answer	Mark	Partial Marks
	Use correct trig formulae to obtain an equation in tan θ or equivalent (e.g all in sin θ or all in cos θ)	*M1	$\frac{1-\tan^2\theta}{2\tan\theta}=2\tan\theta$. Allow $\frac{\cot^2\theta-1}{2\cot\theta}=\frac{2}{\cot\theta}$
	Obtain a correct simplified equation	A1	$5\tan^2\theta=1$ or $\sin^2\theta=\frac{1}{6}$ or $\cos^2\theta=\frac{5}{6}$
	Solve for θ	DM1	Dependent on the first M1
	Obtain answer 24.1° (or 155.9°)	A1	One correct in range to at least 3 sf
	Obtain second answer	A1	**FT** 180° – *their* 24.1° and no others in range. Correct to at least 3 sf. Accept 156° but not 156.0 Ignore values outside range If working in $\tan\theta$ or $\cos\theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1
		5	

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90. 9709_w19_MS_32 Q: 4

	Answer	Mark	Partial Marks
(i)	State $R=\sqrt{7}$	B1	
	Use correct trig formulae to find α	M1	e.g. $\tan\alpha=\frac{1}{\sqrt{6}}$, $\sin\alpha=\frac{1}{\sqrt{7}}$, or $\cos\alpha=\frac{\sqrt{6}}{\sqrt{7}}$
	Obtain $\alpha=22.208°$	A1	ISW
		3	
(ii)	Evaluate $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$ to at least 1 d.p.	B1FT	49.107° to 3 d.p. B1 can be implied by correct answer(s) later. The FT is on *their* R
			SC: allow B1 for a correct alternative equation e.g. $3\tan^2\theta-2\sqrt{6}\tan\theta+1=0$
	Use correct method to find a value of θ in the interval	M1	Must get to θ
	Obtain answer, e.g. 13.4°	A1	Accept correct over-specified answers. 13.449…, 54.3425…
	Obtain second answer, e.g. 54.3° and no extras in the given interval	A1	Ignore answers outside the given interval.
		4	

91. 9709_s18_MS_31 Q: 2

	Answer	Mark
(i)	Use trig formulae and obtain an equation in sin x and cos x	**M1***
	Obtain a correct equation in any form	**A1**
	Substitute exact trig ratios and obtain an expression for tan x	**M1(dep*)**
	Obtain answer $\tan x = \dfrac{-\left(6+\sqrt{6}\right)}{\left(6-\sqrt{2}\right)}$ or equivalent	**A1**
		4
(ii)	State answer, e.g. 118.5°	**B1**
	State second answer, e.g. 298.5°	**B1ft**
		2

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92. 9709_s18_MS_32 Q: 2

	Use correct tan ($A \pm B$) formula and obtain an equation in tan θ	**M1**	$\frac{1}{\tan\theta}+\frac{1-\tan\theta\tan 45}{\tan\theta+\tan 45}=2$ Allow M1 with tan 45° $=\frac{1}{\tan\theta}+\frac{1-\tan\theta}{\tan\theta+1}$
	Obtain a correct equation in any form	**A1**	With values substituted
	Reduce to $3\tan^2\theta = 1$, or equivalent	**A1**	
	Obtain answer $x = 30°$	**A1**	One correct solution
	Obtain answer $x = 150°$	**A1**	Second correct solution and no others in range
	OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ M1A1		
	Reduce to $\tan^2\theta = \frac{1}{3}, \sin^2\theta = \frac{1}{4}, \cos^2\theta = \frac{3}{4}$ or $\cot^2\theta = 3$ A1 etc.		
		5	

93. 9709_s18_MS_33 Q: 5

(i)	Attempt cubic expansion and equate to 1	**M1**
	Obtain a correct equation	**A1**
	Use Pythagoras and double angle formula in the expansion	**M1**
	Obtain the given result correctly	**A1**
	Total:	**4**
(ii)	Use the identity and carry out a method for finding a root	**M1**
	Obtain answer 20.9°	**A1**
	Obtain a second answer, e.g. 69.1°	**A1FT**
	Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval	**A1FT**
	Total:	**4**

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94. 9709_w18_MS_31 Q: 6

	Answer	Mark	Partial Marks
(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**	
	State $R = 2$	**B1**	
	Use trig formulae to obtain α	**M1**	
	Obtain α = 30° with no errors seen	**A1**	
		4	

	Answer	Mark	Partial Marks
(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**	
	Carry out a correct method to find a value of x in the given interval	**M1**	
	Obtain answer x = 75°	**A1**	
	Obtain a second answer e.g. x = 165° and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**	
		4	

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95. 9709_w18_MS_32 Q: 2

	Answer	Mark	Partial Marks
	Use trig formula and obtain an equation in $\sin\theta$ and $\cos\theta$	**M1***	Condone sign error in expansion and/or omission of "$+\cos\theta$" $\sin\theta\cos 30° - \cos\theta\sin 30° + \cos\theta = 2\sin\theta$
	Obtain an equation in $\tan\theta$	**M1(dep*)**	e.g. $\tan\theta = \frac{1-\sin 30°}{2-\cos 30°}$ Can be implied by correct answer following correct expansion. Otherwise need to see working
	Obtain $\tan\theta = 1/\left(4-\sqrt{3}\right)$, or equivalent	**A1**	$\frac{4+\sqrt{3}}{13}$, 0.4409.... (2 s.f or better)
	Obtain final answer $\theta = 23.8°$ and no others in range	**A1**	At least 3 sf (23.7939....) ignore extra values outside range
		4	

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96. 9709_m17_MS_32 Q: 4

	Answer	Mark
(i)	State $R = 17$	**B1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 61.93°$ with no errors seen	**A1**
	Total:	**3**
(ii)	Evaluate $\cos^{-1}(4/17)$ to at least 1d.p. (76.39° to 2 d.p.)	**B1**√
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer, e.g. $x = 7.2°$	**A1**
	Obtain second answer, e.g. $x = 110.8°$and no others	**A1**
	[Ignore answers outside the given interval.]	
	[Treat answers in radians as a misread.]	
	Total:	**4**

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97. 9709_s17_MS_31 Q: 8

	Answer	Mark
(i)	Use $\sin(A-B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	**M1**
	Collect terms and reach $\sqrt{3}\sin x - 2\cos x$, or equivalent	**A1**
	Obtain $R = \sqrt{7}$	**A1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 49.11°$ with no errors seen	**A1**
	Total:	**5**

	Answer	Mark
(ii)	Evaluate $\sin^{-1}(1/\sqrt{7})$ to at least 1 d.p. (22.21° to 2 d.p.)	**B1 FT**
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer 71.3°	**A1**
	[ignore answers outside given range.]	
	Total:	**3**

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98. 9709_s17_MS_32 Q: 3

	Answer	Mark
(i)	Use correct formulae to express the equation in terms of $\cos\theta$ and $\sin\theta$	**M1**
	Use Pythagoras and express the equation in terms of $\cos\theta$ only	**M1**
	Obtain correct 3-term equation, e.g. $2\cos^4\theta + \cos^2\theta - 2 = 0$	**A1**
	Total:	**3**
(ii)	Solve a 3-term quadratic in $\cos^2\theta$ for $\cos\theta$	**M1**
	Obtain answer $\theta = 152.1°$ only	**A1**
	Total:	**2**

99. 9709_s17_MS_33 Q: 1

	Answer	Mark
	Express the LHS in terms of either cos x and sin x or in terms of tan x	**B1**
	Use Pythagoras	**M1**
	Obtain the given answer	**A1**
	Total:	**3**

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100. 9709_w17_MS_31 Q: 4

	Answer	Mark
(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of tan x	**M1**
	Using $\tan 45° = 1$ express LHS as a single fraction	**A1**
	Use Pythagoras or correct double angle formula	**M1**
	Obtain given answer	**A1**
		4
(ii)	Show correct sketch for one branch	**B1**
	Both branches correct and nothing else seen in the interval	**B1**
	Show asymptote at $x = 45°$	**B1**
		3

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101. 9709_w17_MS_32 Q: 3

	Answer	Mark
	Use correct $\tan(A \pm B)$ formula and express LHS in terms of tan θ	**M1**
	Using $\tan 60° = \sqrt{3}$ and $\cot\theta = 1/\tan\theta$, obtain a correct equation in $\tan\theta$ in any form	**A1**
	Reduce the equation to one in $\tan^2\theta$ only	**M1**
	Obtain $11\tan^2\theta = 1$, or equivalent	**A1**
	Obtain answer 16.8°	**A1**
		5

102. 9709_m16_MS_32 Q: 2

Answer	Mark	
Use tan($A \pm B$) formula and obtain an equation in tan θ	**M1**	
Using $\tan 45° = 1$, obtain a horizontal equation in tan θ in any correct form	**A1**	
Reduce the equation to $7\tan^2\theta - 2\tan\theta - 1 = 0$, or equivalent	**A1**	
Solve a 3-term quadratic for tan θ	**M1**	
Obtain a correct answer, e.g. $\theta = 28.7°$	**A1**	
Obtain a second answer, e.g. $\theta = 165.4°$, and no others	**A1**	[6]
[Ignore answers outside the given interval. Treat answers in radians as a misread (0.500, 2.89).]		

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103. 9709_s16_MS_31 Q: 3

Answer	Mark
Correctly restate the equation in terms of sin θ and cos θ	**B1**
Using Pythagoras obtain a horizontal equation in cos θ	**M1**
Reduce the equation to a correct quadratic in cos θ, e.g. $3\cos^2\theta - \cos\theta - 2 = 0$	**A1**
Solve a 3-term quadratic for cos θ	**M1**
Obtain answer θ = 131.8° only	**A1**
	[5]
[Ignore answers outside the given interval.]	

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104. 9709_s16_MS_32 Q: 5

	Answer	Mark	
(i)	*EITHER*: Express cos 4θ in terms of cos 2θ and/or sin 2θ	**B1**	
	Use correct double angle formulae to express LHS in terms of sin θ and/or cos θ	**M1**	
	Obtain a correct expression in terms of sin θ alone	**A1**	
	Reduce correctly to the given form	**A1**	
	OR: Use correct double angle formula to express RHS in terms of cos 2θ	**M1**	
	Express $\cos^2 2\theta$ in terms of cos 4θ	**B1**	
	Obtain a correct expression in terms of cos 4θ and cos 2θ	**A1**	
	Reduce correctly to the given form	**A1**	[4]
(ii)	Use the identity and carry out a method for finding a root	**M1**	
	Obtain answer 68.5°	**A1**	
	Obtain a second answer, e.g. 291.5°	**A1**√	
	Obtain the remaining answers, e.g. 111.5° and 248.5°, and no others in the given interval	**A1**√	[4]
	[Ignore answers outside the given interval. Treat answers in radians as a misread.]		

105. 9709_s16_MS_33 Q: 3

	Answer	Mark
(i)	State answer R =3	**B1**
	Use trig formula to find	**M1**
	Obtain $\alpha = 41.81°$ with no errors seen	**A1**
		[3]
(ii)	Evaluate $\cos^{-1}(0.4)$ to at least 1 d.p. (66.42°to 2 d.p.)	**B1**√
	Carry out an appropriate method to find a value of x in the given range	**M1**
	Obtain answer 216.5° only	**A1**
	[Ignore answers outside the given interval.]	[3]

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106. 9709_w16_MS_31 Q: 3

		Answer	Mark	
	EITHER:	Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$	**B1**	
		Correct method to obtain a horizontal equation in $\sin\theta$	**M1**	
		Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2\theta - \sin\theta - 2 = 0$	**A1**	
		Solve a three-term quadratic for $\sin\theta$	**M1**	
		Obtain final answer $\theta = -41.8°$ only	**A1**	
		[Ignore answers outside the given interval.]		
	OR 1:	Square both sides of the equation and use $1 + \tan^2\theta = \sec^2\theta$	**B1**	
		Correct method to obtain a horizontal equation in $\sin\theta$	**M1**	
		Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2\theta - 6\sin\theta - 8 = 0$	**A1**	
		Solve a three-term quadratic for $\sin\theta$	**M1**	
		Obtain final answer $\theta = -41.8°$ only	**A1**	
	OR 2:	Multiply through by ($\sec\theta + \tan\theta$)	**M1**	
		Use $\sec^2\theta - \tan^2\theta = 1$	**B1**	
		Obtain $1 = 3 + 3\sin\theta$	**A1**	
		Solve for $\sin\theta$	**M1**	
		Obtain final answer $\theta = -41.8°$ only	**A1**	[5]

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107. 9709_w16_MS_33 Q: 3

	Answer	Mark	
	Use the tan 2A formula to obtain an equation in tan θ only	**M1**	
	Obtain a correct horizontal equation	**A1**	
	Rearrange equation as a quadratic in tan θ, e.g. $3\tan^2\theta + 2\tan\theta - 1 = 0$	**A1**	
	Solve for θ (usual requirements for solution of quadratic)	**M1**	
	Obtain answer, e.g. 18.4°	**A1**	
	Obtain second answer, e.g. 135° , and no others in the given interval	**A1**	[6]

108. 9709_s15_MS_32 Q: 4

	Answer	Mark	
(i)	State $R = \sqrt{13}$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = 33.69°$ with no errors seen	A1	**[3]**
(ii)	Evaluate $\sin^{-1}(1/\sqrt{13})$ to at least 1 d.p. (16.10° to 2 d.p)	B1✓	
	Carry out an appropriate method to find a value of θ in the interval $0° < \theta < 180°$	M1	
	Obtain answer $\theta = 130.2°$ and no other in the given interval [Ignore answers outside the given interval.] [Treat answers in radians as a misread and deduct A1 from the marks for the angles.]	A1	**[3]**

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109. 9709_s15_MS_33 Q: 3

Answer	Mark	
Use correct tan 2*A* and cot *A* formulae to form an equation in tan *x*	M1	
Obtain a correct equation in any form	A1	
Reduce equation to the form $\tan^2 x + 6\tan x - 3 = 0$, or equivalent	A1	
Solve a three term quadratic in tan *x* for *x*, **as in Q1.**	M1	
Obtain answer, e.g. 24.9° (24.896)	A1	
Obtain second answer, e.g. 98.8 (98.794) and no others in the given interval [Ignore outside the given interval. Treat answers in radians as a misread.] Radian answers 0.43452, 1.7243	A1	**6**

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110. 9709_w15_MS_31 Q: 3

Answer	Mark	
Use $\tan(A \pm B)$ and obtain an equation in tan θ and tan ϕ	**M1***	
Substitute throughout for tan θ or for tan ϕ	**dep M1***	
Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent	**A1**	
Solve a 3-term quadratic and find an angle	**M1**	
Obtain answer $\theta = 135°$, $\phi = 63.4°$	**A1**	
Obtain answer $\theta = 53.1°$, $\phi = 161.6°$ [Treat answers in radians as a misread. Ignore answers outside the given interval.] [SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]	**A1**	[6]

111. 9709_w15_MS_33 Q: 6

Answer	Mark	
State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$	**B1**	
Divide by $\cos A$ to find value of $\tan A$	**M1**	
Obtain $\tan A = 3$	**A1**	
Use identity $\sec^2 B = 1 + \tan^2 B$	**B1**	
Solve three-term quadratic equation and find $\tan B$	**M1**	
Obtain $\tan B = \frac{3}{2}$ only	**A1**	
Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$	**M1**	
Obtain $\frac{3}{11}$	**A1**	[8]

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112. 9709_s20_MS_31 Q: 4

(a)	Use product rule	**M1**
	Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$	**A1**
	Equate derivative to zero and obtain an equation in one trigonometric ratio	**M1**
	Obtain $x = 1.43$ only	**A1**
		4
(b)	Use a correct method to determine the nature of the stationary point e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$ $x = 1.44,\ y' = -0.07e^{2.88} < 0$	**M1**
	Show that it is a maximum point	**A1**
		2

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113. 9709_s20_MS_32 Q: 4

	Use correct product rule	**M1**
	Obtain correct derivative in any form, e.g. $-\sin x \sin 2x + 2\cos x \cos 2x$	**A1**
	Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$	**M1**
	Equate derivative to zero and obtain an equation in one trig function	**M1**
	Obtain $3\sin 2x = 1$, or $3\cos 2x = 2$ or $2\tan 2x = 1$	**A1**
	Solve and obtain $x = 0.615$	**A1**
		6

114. 9709_s20_MS_33 Q: 4

(a)	Use the product rule	**M1**
	State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent	**M1**
	Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right)+\frac{2x}{x^2+4}$, or equivalent	**A1**
		3
(b)	State or imply y-coordinate is $\frac{1}{2}\pi$	**B1**
	Carry out a complete method for finding p, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\frac{\frac{1}{2}\pi - p}{2}$	**M1**
	Obtain answer $p = -1$	**A1**
		3

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115. 9709_w20_MS_31 Q: 3

	Answer	Mark	Partial Marks
	State or imply $\frac{dx}{d\theta} = 2\sin 2\theta$ or $\frac{dy}{d\theta} = 2 + 2\cos 2\theta$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
	Obtain correct answer $\frac{dy}{dx} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	AG. Must have simplified numerator in terms of cosθ.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	**M1**	
	$\frac{dx}{d\theta}$ or $\frac{dy}{d\theta}$	**B1**	
	$\frac{dy}{dx} = \frac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	**M1 A1**	
	Simplify to given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	AG
	Alternative method for question 3		
	Set $= 2\theta$. State $\frac{dx}{dt} = \sin t$ or $\frac{dy}{dt} = 1 + \cos t$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain correct answer $\frac{dy}{dx} = \frac{1+\cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	
		5	

116. 9709_w20_MS_32 Q: 5

	Answer	Mark	Partial Marks
(a)	State $\frac{dx}{d\theta} = \sec^2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta\cos\theta$	**B1**	CWO, AEF.
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
	Alternative method for question 5(a)		
	Convert to Cartesian form and differentiate	**M1**	$y = \frac{1}{1+x^2}$
	$\frac{dy}{dx} = \frac{-2x}{\left(1+x^2\right)^2}$	**A1**	OE
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
		3	
(b)	Use correct product rule to obtain $\frac{d}{d\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$	**M1**	Condone incorrect naming of the derivative For work done in correct context
	Obtain correct derivative in any form	**A1**	e.g. $\pm\left(-2\cos^4\theta + 6\sin^2\theta\cos^2\theta\right)$
	Equate derivative to zero and obtain an equation in one trig ratio	**A1**	e.g. $3\tan^2\theta = 1$, or $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**	Or $-\frac{\sqrt{3}}{3}$
	Alternative method for question 5(b)		
	Use correct quotient rule to obtain $\frac{d^2y}{dx^2}$	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{-2\left(1+x^2\right)^2 + 2\times 2x\times 2x\left(1+x^2\right)}{\left(1+x^2\right)^4}$
	Equate derivative to zero and obtain an equation in x^2	**A1**	e.g. $6x^2 = 2$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**	
		4	

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117. 9709_m19_MS_32 Q: 5

	Answer	Mark	Partial Marks
	State $\cos y\frac{dy}{dx}$ as derivative of $\sin y$	**B1**	
	State correct derivative in terms of x and y, e.g. $\sec^2 x / \cos y$	**B1**	
	State correct derivative in terms of x, e.g. $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$	**B1**	
	Use double angle formula	**M1**	
	Obtain the given answer correctly	**A1**	
		5	

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118. 9709_m19_MS_32 Q: 10

	Answer	Mark	Partial Marks
(i)	State or imply du = – sin x dx	**B1**	
	Using Pythagoras express the integral in terms of u	**M1**	
	Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$	**A1**	
	Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}}+\frac{2}{7}u^{\frac{7}{2}}$, or equivalent	**A1**	
	Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}}+bu^{\frac{7}{2}}$	**M1**	Or substitute original limits correctly in an integral of the form $a(\cos x)^{\frac{3}{2}}+b(\cos x)^{\frac{7}{2}}$
	Obtain answer $\frac{8}{21}$	**A1**	
		6	
(ii)	Use product rule and chain rule at least once	**M1**	
	Obtain correct derivative in any form	**A1 + A1**	
	Equate derivative to zero and obtain a horizontal equation in integral powers of sin x and cos x	**M1**	
	Use correct methods to obtain an equation in one trig function	**M1**	
	Obtain $\tan^2 x=6$, $7\cos^2 x=1$ or $7\sin^2 x=6$, or equivalent, and obtain answer 1.183	**A1**	
		6	

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119. 9709_s19_MS_31 Q: 3

	Answer	Mark	Partial Marks
	State or imply $3y^2+6xy\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $3xy^2$	**B1**	
	State or imply $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^3	**B1**	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	**M1**	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x^2+y^2}{y^2-2xy}\right)$ For incorrect derivative need to see the substitution
	Obtain final answer $\frac{10}{3}$ or equivalent	**A1**	3.33 or better. Allow $\frac{30}{9}$ ISW after correct answer seen
		4	

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120. 9709_s19_MS_32 Q: 4

	Answer	Mark	Partial Marks
	Use correct quotient rule	M1	Allow use of correct product rule on $x\times(1+\ln x)^{-1}$
	Obtain correct derivative in any form	A1	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(1+\ln x)-x\times\frac{1}{x}}{(1+\ln x)^2}=\left(\frac{1}{1+\ln x}-\frac{1}{(1+\ln x)^2}\right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in $\ln x$ or $(1+\ln x)$	M1	Horizontal form. Accept $\ln x=\frac{1}{4}(1+\ln x)^2$
	Reduce to $(\ln x)^2-2\ln x+1=0$	A1	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in $\ln x$ for x	M1	Must see working if solving incorrect quadratic
	Obtain answer $x=\mathrm{e}$	A1	Accept e^1
	Obtain answer $y=\frac{1}{2}\mathrm{e}$	A1	Exact only with no decimals seen before the exact value. Accept $\frac{\mathrm{e}^1}{2}$ but not $\frac{\mathrm{e}}{1+\ln \mathrm{e}}$
		7	

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121. 9709_s19_MS_32 Q: 10

	Answer	Mark	Partial Marks
(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	B1	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	M1	
	Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly	A1	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$	B1 B1	
	Substitute limits $x=0$ and $x=\frac{1}{3}\pi$ correctly	M1	In their expression
	Obtain answer $\frac{9}{16}$	A1	From correct working seen.
		4	
(iii)	State correct derivative $2\cos 4x+\cos 2x$	B1	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	M1	
	Obtain $4\cos^2 2x+\cos 2x-2=0$	A1	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x=\frac{-1\pm\sqrt{33}}{8}\right)$	M1	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x=\ldots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x-14\cos^2 x+1=0$
	Obtain answer $x=1.29$ only	A1	
		5	

122. 9709_s19_MS_33 Q: 4

	Answer	Mark	Partial Marks
(i)	Use the quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Reduce to $-\dfrac{2e^{-x}}{\left(1-e^{-x}\right)^2}$, or equivalent, and explain why this is always negative	A1	
		3	
(ii)	Equate derivative to – 1 and obtain the given equation	B1	
	State or imply $u^2 - 4u + 1 = 0$, or equivalent in e^a	B1	
	Solve for a	M1	
	Obtain answer $a = \ln\left(2+\sqrt{3}\right)$ and no other	A1	
		4	

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123. 9709_s19_MS_33 Q: 7

	Answer	Mark	Partial Marks
(i)	Use product rule	M1	
	Obtain correct derivative in any form	A1	
		2	
(ii)	Equate derivative to zero and use correct cos($A + B$) formula	M1	
	Obtain the given equation	A1	
		2	
(iii)	Use correct method to solve for x	M1	
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	A1	
	Obtain second answer, e.g. $\frac{7}{12}\pi$, and no other	A1	
		3	

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124. 9709_w19_MS_31 Q: 3

	Answer	Mark	Partial Marks
	State $\frac{dx}{dt} = 2 + 2\cos 2t$	B1	
	Use the chain rule to find the derivative of y	M1	
	Obtain $\frac{dy}{dt} = \frac{2\sin 2t}{1-\cos 2t}$	A1	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain $\frac{dy}{dx} = \operatorname{cosec} 2t$ correctly	A1	AG
		5	

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125. 9709_w19_MS_32 Q: 2

	Answer	Mark	Partial Marks
	Use correct quotient rule or correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2\mathrm{e}^{-2x}\left(1-x^2\right)+2x\mathrm{e}^{-2x}}{\left(1-x^2\right)^2}$
	Equate derivative to zero and obtain a 3 term quadratic in x	**M1**	
	Obtain a correct 3-term equation e.g. $2x^2+2x-2=0$ or $x^2+x=1$	**A1**	From correct work only
	Solve and obtain x = 0.618 only	**A1**	From correct work only
		5	

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126. 9709_w19_MS_32 Q: 5

	Answer	Mark	Partial Marks
	State $4xy+2x^2\frac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent, as derivative of $2x^2y$	**B1**	
	State $y^2+2xy\frac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent, as derivative of xy^2	**B1**	
	Equate attempted derivative of LHS to zero and set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or set numerator equal to zero)	***M1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y^2-4xy}{2x^2-2xy}$
	Reject $y=0$	**B1**	Allow from $y^2-kxy=0$
	Obtain $y=4x$	**A1**	OE from correct numerator. ISW
	Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	**DM1**	$8x^3-16x^3=a^3$ or $\frac{y^3}{8}-\frac{y^3}{4}=a^3$
	Obtain $y=-2a$	**A1**	With no errors seen
		7	
	Alternative method for question 5		
	Rewrite as $y=\frac{a^3}{2x^2-xy}$ and differentiate	**M1**	Correct use of function of a function and implicit differentiation
	Obtain correct derivative (in any form)	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-a^3\left(4x-y-x\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\left(2x^2-xy\right)^2}$
	set $\frac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or set numerator equal to zero)	***M1**	
	Obtain $4x-y=0$	**A1**	
	Confirm $2x^2-xy\neq 0$	**B1**	$x=0$ and $2x=y$ both give $a=0$
	Obtain an equation in y (or in x) and solve for y (or for x)	**DM1**	$8x^3-16x^3=a^3$ or $\frac{y^3}{8}-\frac{y^3}{4}=a^3$
	Obtain $y=-2a$	**A1**	With no errors seen
		7	

127. 9709_w19_MS_33 Q: 4

	Answer	Mark	Partial Marks
(i)	Use tan $(A + B)$ formula to express the LHS in terms of tan $2x$ and tan x	M1	
	Using the tan $2A$ formula, express the entire equation in terms of tan x	M1	
	Obtain a correct equation in tan x in any form	A1	
	Obtain the given form correctly	A1	AG
		4	
(ii)	Use correct method to solve the given equation for x	M1	
	Obtain answer, e.g. $x = 26.8°$	A1	
	Obtain second answer, e.g. $x = 73.7°$ and no other	A1	Ignore answers outside the given interval
		3	

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128. 9709_m18_MS_32 Q: 5

	Answer	Mark
(i)	State correct derivative of x or y with respect to t	B1
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4\sin 2t}{2 + 2\cos 2t}$, or equivalent	A1
	Use double angle formulae throughout	M1
	Obtain the given answer correctly AG	A1
		5
(ii)	State or imply $t = \tan^{-1}\left(-\frac{1}{4}\right)$	B1
	Obtain answer $x = -0.961$	B1
		2

129. 9709_s18_MS_31 Q: 3

	Answer	Mark
	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a\sin x = b$	**M1 ***
	Solve for x	**M1(dep*)**
	Obtain answer 0.340	**A1**
	Obtain second answer 2.802 and no other in the given interval	**A1**
		6

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130. 9709_s18_MS_32 Q: 5

	Answer	Mark	Partial Marks
(i)	State or imply $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^3	**B1**	
	State or imply $6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ as derivative of $x^2(x+3y)$	**B1**	$3x^2+6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}-3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=0$
	Equate derivative of the LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Given answer so check working carefully
	Obtain the given answer	**A1**	
		4	
(ii)	Equate derivative to – 1 and solve for y	**M1***	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	**M1(dep*)**	
	Obtain answer $(1, -2)$	**A1**	
	Obtain answer $(\sqrt[3]{3}, 0)$	**B1**	Must be exact e.g. $e^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

131. 9709_s18_MS_33 Q: 8

	Answer	Mark
(i)	State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	**B1**
	State or imply $3y^2 + 6xy \dfrac{dy}{dx}$ as derivative of $3xy^2$	**B1**
	Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
	Total:	**4**
(ii)	Equate denominator to zero and solve for y	**M1***
	Obtain $y = 0$ and $x = a$	**A1**
	Obtain $y = ax$ and substitute in curve equation to find x or to find y	**M1(dep*)**
	Obtain $x = -a$	**A1**
	Obtain $y = 2a$	**A1**
	Total:	**5**

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132. 9709_w18_MS_31 Q: 4

	Answer	Mark	Partial Marks
(i)	Obtain $\dfrac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\dfrac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	**B1**	
	Use $dy/dx = dy/d\theta \div dx/d\theta$	**M1**	
	Obtain correct $\dfrac{dy}{dx}$ in any form, e.g. $-\dfrac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	**A1**	
		3	
(ii)	Equate denominator to zero and use any correct double angle formula	**M1***	
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	**A1**	
	Solve for θ	**depM1***	
	Obtain $x = 3\sqrt{3}/2$ and $y = \dfrac{1}{2}$, or exact equivalents	**A1**	
		4	

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133. 9709_m17_MS_32 Q: 3

	Answer	Mark
(i)	Sketch a relevant graph, e.g. $y = e^{-\frac{1}{2}x}$	**B1**
	Sketch a second relevant graph, e.g. $y = 4 - x^2$, and justify the given statement	**B1**
	Total:	**2**
(ii)	Calculate the value of a relevant expression or values of a pair of expressions at $x = -1$ and $x = -1.5$	**M1**
	complete the argument correctly with correct calculated values	**A1**
	Total:	**2**

	Answer	Mark
(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer – 1.41	**A1**
	Show sufficient iterations to 4 d.p. to justify – 1.41 to 2 d.p., or show there is a sign change in the interval (– 1.415, – 1.405)	**A1**
	Total:	**3**

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134. 9709_m17_MS_32 Q: 5

	Answer	Mark
	Use product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$	**M1**
	Obtain $\tan^2 x - a\tan x + 1 = 0$, or equivalent	**A1**
	Use the condition for a quadratic to have only one root	**M1**
	Obtain answer $a = 2$	**A1**
	Obtain answer $x = \frac{1}{4}\pi$	**A1**
	Total:	**7**

135. 9709_s17_MS_31 Q: 4

	Answer	Mark
(i)	Use chain rule to differentiate x $\left(\frac{dx}{d\theta}=-\frac{\sin\theta}{\cos\theta}\right)$	M1
	State $\frac{dy}{d\theta}=3-\sec^2\theta$	B1
	Use $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$	M1
	Obtain correct $\frac{dy}{dx}$ in any form e.g. $\frac{3-\sec^2\theta}{-\tan\theta}$	A1
	Obtain $\frac{dy}{dx}=\frac{\tan^2\theta-2}{\tan\theta}$, or equivalent	A1
	Total:	**5**
(ii)	Equate gradient to -1 and obtain an equation in $\tan\theta$	M1
	Solve a 3 term quadratic $(\tan^2\theta+\tan\theta-2=0)$ in $\tan\theta$	M1
	Obtain $\theta=\frac{\pi}{4}$ and $y=\frac{3\pi}{4}-1$ only	A1
	Total:	**3**

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136. 9709_s17_MS_32 Q: 4

	Answer	Mark
(i)	State $\frac{dy}{dt}=4+\frac{2}{2t-1}$	B1
	Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	M1
	Obtain answer $\frac{dy}{dx}=\frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t}+\frac{2}{4t^2-2t}$	A1
	Total:	**3**
(ii)	Use correct method to find the gradient of the normal at $t=1$	M1
	Use a correct method to form an equation for the normal at $t=1$	M1
	Obtain final answer $x+3y-14=0$, or horizontal equivalent	A1
	Total:	**3**

137. 9709_s17_MS_33 Q: 5

	Answer	Mark
(i)	Use the chain rule	**M1**
	Obtain correct derivative in any form	**A1**
	Use correct trigonometry to express derivative in terms of $\tan x$	**M1**
	Obtain $\frac{dy}{dx} = -\frac{4\tan x}{4+\tan^2 x}$, or equivalent	**A1**
	Total:	**4**
(ii)	Equate derivative to –1 and solve a 3–term quadratic for tan x	**M1**
	Obtain answer x=1.11 and no other in the given interval	**A1**
	Total:	**2**

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138. 9709_w17_MS_31 Q: 5

	Answer	Mark
(i)	State or imply $y^3 + 3xy^2\frac{dy}{dx}$ as derivative of xy^3	**B1**
	State or imply $4y^3\frac{dy}{dx}$ as derivative of y^4	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
		4
(ii)	Equate numerator to zero	***M1**
	Obtain $y = -2x$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1**
	Obtain final answer $x = -1, y = 2$ and $x = 1, y = -2$	**A1**
		4

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139. 9709_w17_MS_32 Q: 4

	Answer	Mark
(i)	Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate	M1
	Obtain correct derivative in any form	A1
	Equate the derivative to zero and solve for x	M1
	Obtain $x = \frac{1}{6}\pi$	A1
	Obtain $y = \sqrt{3}$	A1
		5
(ii)	Carry out an appropriate method for determining the nature of a stationary point	M1
	Show the point is a minimum point with no errors seen	A1
		2

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140. 9709_w17_MS_32 Q: 6

	Answer	Mark
(i)	State or imply $3x^2y + x^3\frac{dy}{dx}$ as derivative of x^3y	B1
	State or imply $9xy^2\frac{dy}{dx} + 3y^3$ as derivative of $3xy^3$	B1
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer AG	A1
		4
(ii)	Equate numerator to zero and use $x = -y$ to obtain an equation in x or in y	M1
	Obtain answer $x = a$ and $y = -a$	A1
	Obtain answer $x = -a$ and $y = a$	A1
	Consider and reject $y = 0$ and $x = y$ as possibilities	B1
		4

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141. 9709_m16_MS_32 Q: 6

		Answer	Mark	
(i)	*EITHER*:	State correct derivative of $\sin y$ with respect to x	**B1**	
		Use product rule to differentiate the LHS	**M1**	
		Obtain correct derivative of the LHS	**A1**	
		Obtain a complete and correct derived equation in any form	**A1**	
		Obtain a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form	**A1**	
	OR:	State correct derivative of $\sin y$ with respect to x	**B1**	
		Rearrange the given equation as $\sin y = x/(\ln x + 2)$ and attempt to differentiate both sides	**B1**	
		Use quotient or product rule to differentiate the RHS	**M1**	
		Obtain correct derivative of the RHS	**A1**	
		Obtain a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form	**A1**	[5]
(ii)		Equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and obtain a horizontal equation in $\ln x$ or $\sin y$	**M1**	
		Solve for $\ln x$	**M1**	
		Obtain final answer $x = 1/\mathrm{e}$, or exact equivalent	**A1**	[3]

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142. 9709_s16_MS_31 Q: 5

Answer	Mark
Use product rule	**M1**
Obtain correct derivative in any form, e.g. $\cos x \cos 2x - 2\sin x \sin 2x$	**A1**
Equate derivative to zero and use double angle formulae	**M1**
Remove factor of $\cos x$ and reduce equation to one in a single trig function	**M1**
Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$	**A1**
Solve and obtain $x = 0.421$	**A1**
	[6]
[Alternative: Use double angle formula M1.Use chain rule to differentiate M1. Obtain correct derivative e.g. $\cos\theta - 6\sin^2\theta\cos\theta$ A1, then as above.]	

143. 9709_s16_MS_31 Q: 7

	Answer	Mark
(i)	State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$	**B1**
	State $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**
	Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1** [4]
(ii)	Equate numerator to zero	**M1***
	Obtain $x = 2y$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1***
	Obtain the point (–2, –1)	**A1**
	State the point (0, 1.44)	**B1** [5]

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144. 9709_s16_MS_32 Q: 4

Answer	Mark	
State or imply derivative of $(\ln x)^2$ is $\frac{2\ln x}{x}$	**B1**	
Use correct quotient or product rule	**M1**	
Obtain correct derivative in any form, e.g. $\frac{2\ln x}{x^2}-\frac{(\ln x)^2}{x^2}$	**A1**	
Equate derivative (or its numerator) to zero and solve for ln x	**M1**	
Obtain the point (1, 0) with no errors seen	**A1**	
Obtain the point $(e^2,\ 4e^{-2})$	**A1**	[6]

145. 9709_s16_MS_33 Q: 4

	Answer	Mark
(i)	State $\frac{dx}{dt}=1-\sin t$	**B1**
	Use chain rule to find the derivative of y	**M1**
	Obtain $\frac{dy}{dt}=\frac{\cos t}{1+\sin t}$, or equivalent	**A1**
	Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	**M1**
	Obtain the given answer correctly	**A1** [5]
(ii)	State or imply $t=\cos^{-1}(\frac{1}{3})$	**B1**
	Obtain answers x = 1.56 and x = – 0.898	**B1 + B1** [3]

146. 9709_w16_MS_31 Q: 4

			Answer	Mark	
	EITHER:	*EITHER*:	State $2xy+x^2\frac{dy}{dx}$, or equivalent, as derivative of x^2y	**B1**	
			State $6y^2+12xy\frac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	**B1**	
		OR:	Differentiating LHS using correct product rule, state term $xy(1-6\frac{dy}{dx})$, or equivalent	**B1**	
			State term $(y+x\frac{dy}{dx})(x-6y)$, or equivalent	**B1**	
			Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	**M1***	
			Obtain a horizontal equation, e.g. $6y^2-2xy=0$ (from correct work only)	**A1**	
			Explicitly reject $y = 0$ as a possibility $py^2-qxy=0$	**A1**	
			Obtain an equation in x or y	**DM1**	
			Obtain answer $(-3a, -a)$	**A1**	
	OR:	Rearrange to $y=\frac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\frac{9a^3}{x^2(x-6y)^2}\times.....$		**B1**	
		State term $(x-6y)+x(1-6y')$, or equivalent		**B1**	
		Justify division by $x(x-6y)$		**B1**	
		Set $\frac{dy}{dx}$ equal to zero		**M1***	
		Obtain a horizontal equation, e.g. $6y^2-2xy=0$ (from correct work only)		**A1**	
		Obtain an equation in x or y		**DM1**	
		Obtain answer $(-3a, -a)$		**A1**	[7]

147. 9709_w16_MS_33 Q: 2

	Answer	Mark	
	Use correct quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Use Pythagoras to simplify the derivative to $\frac{1}{1+\cos x}$, or equivalent	**A1**	
	Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	**A1**	[4]

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148. 9709_s15_MS_31 Q: 4

Answer	Mark	
Differentiate to obtain form $a\sin 2x + b\cos x$	M1	
Obtain correct $-6\sin 2x + 7\cos x$	A1	
Use identity $\sin 2x = 2\sin x\cos x$	B1	
Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of x	M1	
Obtain 0.623	A1	
Obtain 2.52	A1	
Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$	A1	
Treat answers in degrees as MR – 1 situation		[7]

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149. 9709_s15_MS_32 Q: 3

	Answer	Mark	
EITHER:	Use correct product rule	M1	
	Obtain correct derivative in any form, e.g. $-\sin x\cos 2x - 2\cos x\sin 2x$	A1	
	Use the correct double angle formulae to express derivative in cos x and sin x, or cos $2x$ and sin x	M1	
*OR*1:	Use correct double angle formula to express y in terms of cos x and attempt differentiation	M1	
	Use chain rule correctly	M1	
	Obtain correct derivative in any form, e.g. $-6\cos^2 x\sin x + \sin x$	A1	
*OR*2:	Use correct factor formula and attempt differentiation	M1	
	Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$	A1	
	Use correct trig formulae to express derivative in terms of cos x and sin x, or sin x	M1	
Equate derivative to zero and obtain an equation in one trig function		M1	
Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$		A1	
Obtain answer x = 1.15 (or 65.9°) and no other in the given interval		A1	**[6]**
[Ignore answers outside the given interval.]			
[SR: Solution attempts following the *EITHER* scheme for the first two marks can earn the second and third method marks as follows:			
Equate derivative to zero and obtain an equation in tan $2x$ and tan x		M1	
Use correct double angle formula to obtain an equation in tan x		M1]	

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150. 9709_s15_MS_33 Q: 4

Answer	Mark	
Use correct quotient or product rule	M1	
Obtain correct derivative in any form	A1	
Equate derivative to zero and obtain a horizontal equation	M1	
Carry out complete method for solving an equation of the form $ae^{3x} = b$, or $ae^{5x} = be^{2x}$	M1	
Obtain $x = \ln 2$, or exact equivalent	A1	
Obtain $y = \frac{1}{3}$, or exact equivalent	A1	**6**

151. 9709_s15_MS_33 Q: 5

	Answer	Mark	
(i)	State $\frac{dx}{dt} = -4a\cos^3 t \sin t$, or $\frac{dy}{dt} = 4a\sin^3 t \cos t$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain correct expression for $\frac{dy}{dx}$ in a simplified form	A1	**3**
(ii)	Form the equation of the tangent	M1	
	Obtain a correct equation in any form	A1	
	Obtain the given answer	A1	**3**
(iii)	State the x-coordinate of P or the y-coordinate of Q in any form	B1	
	Obtain the given result correctly	B1	**2**

152. 9709_w15_MS_31 Q: 5

	Answer	Mark	
(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	**B1**	
	Use product or quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Use Pythagoras	**M1**	
	Justify the given form	**A1**	[5]
(ii)	Fully justify the given statement	**B1**	[1]
(iii)	State answer $x = \frac{1}{4}\pi$	**B1**	[1]

153. 9709_w15_MS_33 Q: 3

Answer	Mark	
Use correct quotient rule or equivalent to find first derivative	**M1***	
Obtain $\dfrac{-(1+\tan x)\sec^2 x-\sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent	**A1**	
Substitute $x=\frac{1}{4}\pi$ to find gradient	**dep M1***	
Obtain $-\frac{3}{2}$	**A1**	
Form equation of tangent at $x=\frac{1}{4}\pi$	**M1**	
Obtain $y=-\frac{3}{2}x+1.68$ or equivalent	**A1**	[6]

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154. 9709_s20_MS_31 Q: 5

(a)	Commence division and reach quotient of the form $2x+k$	**M1**
	Obtain quotient $2x-1$	**A1**
	Obtain remainder 6	**A1**
		3
(b)	Obtain terms x^2-x (**FT** on quotient of the form $2x+k$)	**B1FT**
	Obtain term of the form $a\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**
	Obtain term $\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ (**FT** on a constant remainder)	**A1FT**
	Use $x=1$ and $x=3$ as limits in a solution containing a term of the form $a\tan^{-1}(bx)$	**M1**
	Obtain final answer $\frac{1}{\sqrt{3}}\pi+6$, or exact equivalent	**A1**
		5

155. 9709_s20_MS_31 Q: 7

(a)	Use quotient or product rule	**M1**
	Obtain derivative in any correct form e.g. $\frac{-\sin x\left(1+\sin x\right)-\cos x\left(\cos x\right)}{\left(1+\sin x\right)^2}$	**A1**
	Use Pythagoras to simplify the derivative	**M1**
	Justify the given statement	**A1**
		4
(b)	State integral of the form $a\ln(1+\sin x)$	***M1**
	State correct integral $\ln(1+\sin x)$	**A1**
	Use limits correctly	**DM1**
	Obtain answer $\ln\frac{4}{3}$	**A1**
		4

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156. 9709_s20_MS_32 Q: 3

	Commence integration and reach $ax^{\frac{5}{2}}\ln x+b\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	**M1***
	Obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x-\frac{2}{5}\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	**A1**
	Complete the integration and obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x-\frac{4}{25}x^{\frac{5}{2}}$, or equivalent	**A1**
	Use limits correctly, having integrated twice e.g $\frac{2}{5}\times32\ln4-\frac{4}{25}\times32-\left(\frac{2}{5}\times0\right)+\frac{4}{25}$	**DM1**
	Obtain answer $\frac{128}{5}\ln2-\frac{124}{25}$, or exact equivalent	**A1**
		5

157. 9709_s20_MS_32 Q: 6

(a)	Use quotient or product rule	**M1**
	Obtain correct derivative in any form e.g. $\frac{(1+3x^4)-x\times12x^3}{(1+3x^4)^2}$	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain answer 0.577	**A1**
		4
(b)	State or imply $\mathrm{d}u=2\sqrt{3x}\,\mathrm{d}x$, or equivalent	**B1**
	Substitute for x and $\mathrm{d}x$	**M1**
	Obtain integrand $\frac{1}{2\sqrt{3(1+u^2)}}$, or equivalent	**A1**
	State integral of the form $a\tan^{-1}u$ and use limits $u=0$ and $u=\sqrt{3}$ (or $x=0$ and $x=1$) correctly	**M1**
	Obtain answer $\frac{\sqrt{3}}{18}\pi$, or exact equivalent	**A1**
		5

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158. 9709_s20_MS_33 Q: 2

	Commence integration and reach $a(2-x)e^{-2x}+b\int e^{-2x}\,\mathrm{d}x$, or equivalent	**M1***
	Obtain $-\frac{1}{2}(2-x)e^{-2x}-\frac{1}{2}\int e^{-2x}\,\mathrm{d}x$, or equivalent	**A1**
	Complete integration and obtain $-\frac{1}{2}(2-x)e^{-2x}+\frac{1}{4}e^{-2x}$, or equivalent	**A1**
	Use limits correctly, having integrated twice	**DM1**
	Obtain answer $\frac{1}{4}(3-e^{-2})$, or exact equivalent	**A1**
		5

159. 9709_s20_MS_33 Q: 7

(a)	State or imply the form $\frac{A}{2x-1}+\frac{B}{2x+1}$ and use a relevant method to find A or B	**M1**
	Obtain $A = 1$, $B = -1$	**A1**
		2
(b)	Square the result of part (**a**) and substitute the fractions of part (**a**)	**M1**
	Obtain the given answer correctly	**A1**
		2
(c)	Integrate and obtain $-\frac{1}{2(2x-1)}-\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(2x+1)-\frac{1}{2(2x+1)}$, or equivalent	**B3, 2, 1, 0**
	Substitute limits correctly	**M1**
	Obtain the given answer correctly	**A1**
		5

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160. 9709_w20_MS_31 Q: 10

	Answer	**Mark**	**Partial Marks**
(a)	Use correct product or quotient rule	***M1**	$\frac{dy}{dx}=\left(-\frac{1}{2}\right)(2-x)e^{-\frac{1}{2}x}-e^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**DM1**	
	Obtain $x = 4$	**A1**	ISW
	Obtain $y = -2e^{-2}$, or exact equivalent	**A1**	
		5	
(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x}+b\int e^{-\frac{1}{2}x}dx$	***M1**	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x}+4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x}-2\int e^{-\frac{1}{2}x}dx$	**A1**	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	**A1**	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	**A1**	ISW
	Alternative method for question 10(b)		
	$\frac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx}=2e^{-\frac{1}{2}x}-xe^{-\frac{1}{2}x}$	***M1 A1**	
	$\therefore 2xe^{-\frac{1}{2}x}$	**A1**	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	**A1**	ISW
		5	

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161. 9709_w20_MS_32 Q: 9

	Answer	Mark	Partial Marks
(a)	State or imply the form $\frac{A}{3x+2}+\frac{Bx+C}{x^2+4}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A=3$, $B=-1$, $C=3$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain $\ln(3x+2)\ldots$	B1 FT	The FT is on A
	State a term of the form $k\ln\left(x^2+4\right)$.	M1	From $\int\frac{\lambda x}{x^2+4}\mathrm{d}x$
	$\ldots-\frac{1}{2}\ln\left(x^2+4\right)\ldots$	A1 FT	The FT is on B
	$\ldots+\frac{3}{2}\tan^{-1}\frac{x}{2}$	B1 FT	The FT is on C
	Substitute limits correctly in an integral with at least two terms of the form $a\ln(3x+2)$, $b\ln\left(x^2+4\right)$ and $c\tan^{-1}\left(\frac{x}{2}\right)$, and subtract in correct order	M1	Using terms that have been obtained correctly from completed integrals
	Obtain answer $\frac{3}{2}\ln 2+\frac{3}{8}\pi$, or exact 2-term equivalent	A1	
		6	

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162. 9709_m19_MS_32 Q: 4

	Answer	Mark	Partial Marks
	Integrate by parts and reach $ax^{-\frac{1}{2}}\ln x+b\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$	M1*	
	Obtain $-2x^{-\frac{1}{2}}\ln x+2\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$, or equivalent	A1	
	Complete the integration, obtaining $-2x^{-\frac{1}{2}}\ln x-4x^{-\frac{1}{2}}$, or equivalent	A1	
	Substitute limits correctly, having integrated twice	M1(dep*)	
	Obtain the given answer following full and correct working	A1	
		5	

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163. 9709_s19_MS_31 Q: 1

	Answer	Mark	Partial Marks
	State or imply ordinates 3, 2, 0, 4	B1	These and no more Accept in unsimplified form $\left\|2^0-4\right\|$ etc.
	Use correct formula, or equivalent, with $h=1$ and four ordinates	M1	
	Obtain answer 5.5	A1	
		3	

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164. 9709_s19_MS_32 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{2x+1} + \frac{B}{2x+3} + \frac{C}{(2x+3)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain the values $A = 1$, $B = -1$, $C = 3$	**A1 A1 A1**	
	[Mark the form $\frac{A}{2x+1} + \frac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]		Full marks for the three correct constants – do not actually need to see the partial fractions
		5	
(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1)-\frac{1}{2}\ln(2x+3)-\frac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1)+\frac{x}{2x+3}-\frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	**B1 B1 B1**	**FT** on A, B and C.
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	**M1**	value for upper limit – value for lower limit 1 slip in substituting can still score M1 Condone omission of $\ln(1)$
	Obtain the **given answer** following full and correct working	**A1**	Need to see at least one interim step of valid log work. **AG**
		5	

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165. 9709_s19_MS_33 Q: 2

	Answer	Mark	Partial Marks
	Commence integration and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	**M1***	
	Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x \, dx$, or equivalent	**A1**	
	Complete the integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	**A1**	
	Use limits correctly, having integrated twice	**DM1**	
	Obtain given answer correctly	**A1**	
		5	

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166. 9709_s19_MS_33 Q: 3

	Answer	Mark	Partial Marks
(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	**M1**	
	Obtain a correct expression	**A1**	
	Obtain the given answer	**A1**	
		3	
(ii)	State integral of the form $\pm\ln\cos\theta$	**M1***	
	Use correct limits correctly and insert exact values for the trig ratios	**DM1**	
	Obtain a correct expression, e.g. $-\ln\frac{1}{\sqrt{2}} + \ln\frac{\sqrt{3}}{2}$	**A1**	
	Obtain the given answer following full and exact working	**A1**	
		4	

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167. 9709_w19_MS_31 Q: 6

	Answer	Mark	Partial Marks
(i)	Use correct quotient rule	**M1**	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosec}^2 x$ correctly	**A1**	AG
		2	
(ii)	Integrate by parts and reach $ax\cot x + b\int\cot x\,\mathrm{d}x$	***M1**	
	Obtain $-x\cot x + \int\cot x\,\mathrm{d}x$	**A1**	OE
	State $\pm\ln\sin x$ as integral of $\cot x$	**M1**	
	Obtain complete integral $-x\cot x + \ln\sin x$	**A1**	OE
	Use correct limits correctly	**DM1**	$0+0+\frac{\pi}{4}-\ln\frac{1}{\sqrt{2}}$
	Obtain $\frac{1}{4}(\pi+\ln 4)$ following full and exact working	**A1**	AG
		6	

168. 9709_w19_MS_31 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=-1$, $B=3$, $C=2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Allow in the form $\frac{Ax+B}{x^2}+\frac{C}{x+2}$
		5	
(ii)	Integrate and obtain terms $\ln x-\frac{3}{x}+2\ln(x+2)$	**B1FT +** **B1FT +** **B1FT**	The FT is on *A*, *B*, *C*; or on *A*, *D*, *E*.
	Substitute limits correctly in an integral with terms $a\ln x$, $\frac{b}{x}$ and $c\ln(x+2)$, where $abc\neq 0$	**M1**	$-\ln 4-\frac{3}{4}+2\ln 6(+\ln 1)+3-2\ln 3$
	Obtain $\frac{9}{4}$ following full and exact working	**A1**	AG – work to combine or simplify logs is required
		5	

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169. 9709_w19_MS_31 Q: 9

	Answer	Mark	Partial Marks
(i)	Use cos(A + B) formula to express cos3x in terms of trig functions of 2x and x	**M1**	
	Use double angle formulae and Pythagoras to obtain an expression in terms of cos x only	**M1**	
	Obtain a correct expression in terms of cos x in any form	**A1**	
	Obtain $\cos 3x\equiv 4\cos^3 x-3\cos x$	**A1**	AG
		4	
(ii)	Use identity and solve cubic $4\cos^3 x=-1$ for x	**M1**	$\cos x=-0.6299...$
	Obtain answer 2.25 and no other in the interval	**A1**	Accept 0.717π M1A0 for 129.0°
		2	
(iii)	Obtain indefinite integral $\frac{1}{12}\sin 3x+\frac{3}{4}\sin x$	**B1 +** **B1**	
	Substitute limits in an indefinite integral of the form $a\sin 3x+b\sin x$, where $ab\neq 0$	**M1**	$\frac{1}{4}\left[\frac{1}{3}\sin\pi+3\sin\frac{\pi}{3}-\frac{1}{3}\sin\frac{\pi}{2}-3\sin\frac{\pi}{6}\right]$
	Obtain answer $\frac{1}{24}(9\sqrt{3}-11)$, or exact equivalent	**A1**	
	Alternative method for question 9(iii)		
	$\int\cos x(1-\sin^2 x)\mathrm{d}x=\sin x-\frac{1}{3}\sin^3 x(+C)$	**B1 +** **B1**	
	Substitute limits in an indefinite integral of the form $a\sin x+b\sin^3 x$ where $ab\neq 0$	**M1**	$\left(\frac{\sqrt{3}}{2}-\frac{1}{2}-\frac{1}{4}\frac{\sqrt{3}}{2}+\frac{1}{24}\right)$
	Obtain answer $\frac{1}{24}(9\sqrt{3}-11)$, or exact equivalent	**A1**	
		4	

170. 9709_w19_MS_32 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{2x-1}+\frac{Bx+C}{x^2+2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A=4$, $B=-1$, $C=0$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(ii)	Integrate and obtain term $2\ln(2x-1)$	B1FT	The FT is on A. $\frac{1}{2}A\ln(2x-1)$
	Integrate and obtain term of the form $k\ln(x^2+2)$	*M1	From $\frac{nx}{x^2+2}$
	Obtain term $-\frac{1}{2}\ln(x^2+2)$	A1FT	The FT is on B
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab\neq 0$	DM1	$2\ln 9(-2\ln 1)-\frac{1}{2}\ln 27+\frac{1}{2}\ln 3$
	Obtain answer ln 27 after full and correct exact working	A1	ISW
		5	

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171. 9709_w19_MS_33 Q: 8

	Answer	Mark	Partial Marks
(i)	State or imply ordinates 1, 1.2116…, 2.7597...	B1	
	Use correct formula, or equivalent, with $h=0.6$	M1	
	Obtain answer 1.85	A1	
		3	
(ii)	Explain why the rule gives an overestimate	B1	
		1	
(iii)	Differentiate using quotient or chain rule	M1	
	Obtain correct derivative in terms of sin x and cos x	A1	
	Equate derivative to 2, use Pythagoras and obtain an equation in sin x	M1	
	Obtain $2\sin^2 x+\sin x-2=0$	A1	OE
	Solve a 3-term quadratic for x	M1	
	Obtain answer $x=0.896$ only	A1	
		6	

172. 9709_w19_MS_33 Q: 10

	Answer	Mark	Partial Marks
(i)	Use product rule and chain rule at least once	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	**M1**	
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	**A1**	
	Obtain answer $x = 1.26$	**A1**	
		5	
(ii)	Using $\mathrm{d}u = \pm \sin x\,\mathrm{d}x$ express integrand in terms of u and du	**M1**	
	Obtain integrand $\mathrm{e}^u(u^2 - 1)$	**A1**	OE
	Commence integration by parts and reach $a\mathrm{e}^u(u^2 - 1) + b\int u\mathrm{e}^u\,\mathrm{d}u$	***M1**	
	Obtain $\mathrm{e}^u(u^2 - 1) - 2\int u\mathrm{e}^u\,\mathrm{d}u$	**A1**	OE
	Complete integration, obtaining $\mathrm{e}^u(u^2 - 2u + 1)$	**A1**	OE
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	**DM1**	
	Obtain answer $\frac{4}{\mathrm{e}}$, or exact equivalent	**A1**	
		7	

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173. 9709_m18_MS_32 Q: 1

	Answer	Mark
	State or imply ordinates 1, 0.8556…, 0.6501…, 0	**B1**
	Use correct formula, or equivalent, with $h = \frac{1}{12}\pi$ and four ordinates	**M1**
	Obtain answer 0.525	**A1**
		3

174. 9709_m18_MS_32 Q: 3

	Answer	Mark
(i)	State correct expansion of $\cos(3x+x)$ or $\cos(3x-x)$	**B1**
	Substitute in $\frac{1}{2}(\cos 4x+\cos 2x)$	**M1**
	Obtain the given identity correctly **AG**	**A1**
		3
(ii)	Obtain integral $\frac{1}{8}\sin 4x+\frac{1}{4}\sin 2x$	**B1**
	Substitute limits correctly	**M1**
	Obtain the given answer following full, correct and exact working **AG**	**A1**
		3

175. 9709_m18_MS_32 Q: 8

	Answer	Mark
(i)	State or imply the form $\frac{A}{2x+1}+\frac{Bx+C}{x^2+9}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A=3$, $B=1$ and $C=0$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
(ii)	Integrate and obtain term $\frac{3}{2}\ln(2x+1)$ (FT on A value)	**B1 FT**
	Integrate and obtain term of the form $k\ln(x^2+9)$	**M1**
	Obtain term $\frac{1}{2}\ln(x^2+9)$ (FT on B value)	**A1 FT**
	Substitute limits correctly in an integral of the form $a\ln(2x+1)+b\ln(x^2+9)$, where $ab\neq 0$	**M1**
	Obtain answer ln 45 after full and correct working	**A1**
		5

176. 9709_s18_MS_31 Q: 5

	Answer	Mark
(i)	State or imply $dx = -2\cos\theta\sin\theta\, d\theta$, or equivalent	**B1**
	Substitute for x and dx, and use Pythagoras	**M1**
	Obtain integrand $\pm 2\cos^2\theta$	**A1**
	Justify change of limits and obtain given answer correctly	**A1**
		4
(ii)	Obtain indefinite integral of the form $a\theta + b\sin 2\theta$	**M1***
	Obtain $\theta + \frac{1}{2}\sin 2\theta$	**A1**
	Use correct limits correctly	**M1(dep*)**
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	**A1**
		4

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177. 9709_s18_MS_32 Q: 4

	Answer	Mark	Partial Marks
(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	**M1**	$\frac{2\sin x - 2\sin x\cos x}{1-(2\cos^2 x - 1)}$
	Obtain a correct expression	**A1**	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	**M1**	
	Obtain the given RHS correctly *OR (working R to L)*:	**A1**	
	$\frac{\sin x}{1+\cos x} \times \frac{1-\cos x}{1-\cos x} = \frac{\sin x - \sin x\cos x}{1-\cos^2 x}$ M1A1 $= \frac{2\sin x - 2\sin x\cos x}{2-2\cos^2 x}$		Given answer so check working carefully
	$= \frac{2\sin x - \sin 2x}{1-\cos 2x}$ M1A1		
		4	
(ii)	State integral of the form $a\ \ln(1+\cos x)$	**M1***	If they use the substitution $u = 1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	**A1**	
	Substitute correct limits in correct order	**M1(dep)***	
	Obtain answer $\ln\left(\frac{3}{2}\right)$, or equivalent	**A1**	
		4	

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178. 9709_s18_MS_32 Q: 8

	Answer	Mark	Partial Marks
(i)	Use correct product or quotient rule	**M1**	$\frac{dy}{dx}=-\frac{1}{3}(x+1)e^{-\frac{1}{3}x}+e^{-\frac{1}{3}x}$ or $\frac{dy}{dx}=\frac{e^{\frac{1}{3}x}-(x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 2$ with no errors seen	**A1**	
		4	
(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x}+b\int e^{-\frac{1}{3}x}\,dx$	**M1***	
	Obtain $-3(x+1)e^{-\frac{1}{3}x}+3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	**A1**	$-3xe^{-\frac{1}{3}x}+\int 3e^{-\frac{1}{3}x}\,dx-3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x}-9e^{-\frac{1}{3}x}$, or equivalent	**A1**	
	Use **correct** limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice	**M1(dep*)**	
	Obtain answer $9e^{\frac{1}{3}}-12$, or equivalent	**A1**	
		5	

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179. 9709_s18_MS_33 Q: 3

	Answer	Mark
	Integrate by parts and reach $ax\sin 3x+b\int \sin 3x\,dx$	**M1***
	Obtain $\frac{1}{3}x\sin 3x-\frac{1}{3}\int \sin 3x\,dx$, or equivalent	**A1**
	Complete the integration and obtain $\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x$, or equivalent	**A1**
	Substitute limits correctly having integrated twice and obtained $ax \sin 3x + b \cos 3x$	**M1(dep*)**
	Obtain answer $\frac{1}{18}(\pi-2)$ OE	**A1**
	Total:	**5**

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180. 9709_s18_MS_33 Q: 7

	Answer	Mark
(i)	State answer $R = \sqrt{5}$	**B1**
	Use trig formulae to find tan α	**M1**
	Obtain $\tan\alpha = 2$	**A1**
	Total:	**3**
(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	**B1FT**
	State correct indefinite integral $3\tan(\theta - \alpha)$	**B1FT**
	Substitute limits correctly	**M1**
	Use tan($A \pm B$) formula	**M1**
	Obtain the given exact answer correctly	**A1**
	Total:	**5**

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181. 9709_w18_MS_31 Q: 7

	Answer	Mark	Partial Marks
(i)	Use product rule	**M1***	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***	
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**	
	Obtain answer $x = 0.685$	**A1**	
		5	

	Answer	Mark	Partial Marks
(ii)	Use the given substitution and reach $a\int(u^2 - u^4)\,\mathrm{d}u$	**M1**	
	Obtain correct integral with $a = 5$ and limits 0 and 1	**A1**	
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	**M1**	
	Obtain answer $\frac{2}{3}$	**A1**	
		4	

182. 9709_w18_MS_31 Q: 9

	Answer	Mark	Partial Marks
(i)	State or imply the form $\frac{A}{2-x}+\frac{B}{3+2x}+\frac{C}{(3+2x)^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 1$, $B = -1$, $C = 3$	A1	
	Obtain a second value	A1	
	Obtain the third value [Mark the form $\frac{A}{2-x}+\frac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	A1	
		5	
(ii)	Integrate and obtain terms $-\ln(2-x)-\frac{1}{2}\ln(3+2x)-\frac{3}{2(3+2x)}$	B3ft	The f.t is on A, B, C; or on A, D, E.
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	M1	
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	A1	
		5	

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183. 9709_w18_MS_32 Q: 3

	Answer	Mark	Partial Marks
(i)	Integrate by parts and reach $a\frac{\ln x}{x^2}+b\int\frac{1}{x}.\frac{1}{x^2}\,dx$	M1*	
	Obtain $\pm\frac{1}{2}\frac{\ln x}{x^2}\pm\int\frac{1}{x}.\frac{1}{2x^2}\,dx$, or equivalent	A1	
	Complete integration correctly and obtain $-\frac{\ln x}{2x^2}-\frac{1}{4x^2}$, or equivalent	A1	Condone without '$+C$' ISW
		3	

	Answer	Mark	Partial Marks
(ii)	Substitute limits correctly in an expression of the form $a\frac{\ln x}{x^2}+\frac{b}{x^2}$ or equivalent	M1(dep*)	$-\frac{1}{8}\ln 2-\frac{1}{16}+\frac{1}{4}$
	Obtain the **given answer** following full and **exact** working	A1	The step $\ln 2=\frac{1}{2}\ln 4$ or 2ln2 = ln4 needs to be clear.
		2	

184. 9709_w18_MS_32 Q: 7

	Answer	Mark	Partial Marks
(i)	Use correct quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3\sin x\left(2+\sin x\right)-3\cos x\cos x}{\left(2+\sin x\right)^2}$ Condone invisible brackets if recovery implied later.
	Equate numerator to zero	**M1**	
	Use $\cos^2 x+\sin^2 x=1$ and solve for $\sin x$	**M1**	$-6\sin x-3=0 \Rightarrow \sin x=\ldots$
	Obtain coordinates $x=-\pi/6$ and $y=\sqrt{3}$ ISW	**A1 + A1**	From correct working. No others in range
			SR: A candidate who only states the numerator of the derivative, but justifies this, can have full marks. Otherwise they score M0A0M1M1A0A0
		6	
(ii)	State indefinite integral of the form k ln (2 + sin x)	**M1***	
	Substitute limits correctly, equate result to 1 and obtain 3 ln (2 + sin a) – 3 ln 2 = 1	**A1**	or equivalent
	Use correct method to solve for a	**M1(dep*)**	Allow for a correct method to solve an incorrect equation, so long as that equation has a solution. $1+\frac{1}{2}\sin a=\mathrm{e}^{1/3}\Rightarrow a=\sin^{-1}\left[2\left(\mathrm{e}^{1/3}-1\right)\right]$ Can be implied by 52.3°
	Obtain answer a = 0.913 or better	**A1**	Ignore additional solutions. Must be in radians.
		4	

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185. 9709_m17_MS_32 Q: 7

	Answer	Mark
7(i)	State or imply $\frac{\mathrm{d}V}{\mathrm{d}t} = 2\frac{\mathrm{d}h}{\mathrm{d}t}$	B1
	State or imply $\frac{\mathrm{d}V}{\mathrm{d}t} = 1 - 0.2\sqrt{h}$	B1
	Obtain the given answer correctly	B1
	Total:	3
7(ii)	State or imply $\mathrm{d}u = -\frac{1}{2\sqrt{h}}\,\mathrm{d}h$, or equivalent	B1
	Substitute for h and dh throughout	M1
	Obtain $T = \int_3^5 \frac{20(5-u)}{u}\,\mathrm{d}u$, or equivalent	A1
	Integrate and obtain terms $100\ln u - 20u$, or equivalent	A1
	Substitute limits $u = 3$ and $u = 5$ correctly	M1
	Obtain answer 11.1, with no errors seen	A1
	Total:	6

186. 9709_m17_MS_32 Q: 10

	Answer	Mark
(i)	State or imply derivative is $2\dfrac{\ln x}{x}$	**B1**
	State or imply gradient of the normal at $x = \mathrm{e}$ is $-\frac{1}{2}\mathrm{e}$, or equivalent	**B1**
	Carry out a complete method for finding the x-coordinate of Q	**M1**
	Obtain answer $x = \mathrm{e} + \dfrac{2}{\mathrm{e}}$, or exact equivalent	**A1**
	Total:	**4**
(ii)	Justify the given statement by integration or by differentiation	**B1**
	Total:	**1**
(iii)	Integrate by parts and reach $ax(\ln x)^2 + b\int x.\dfrac{\ln x}{x}\,\mathrm{d}x$	**M1***
	Complete the integration and obtain $x(\ln x)^2 - 2x\ln x + 2x$, or equivalent	**A1**
	Use limits $x = 1$ and $x = \mathrm{e}$ correctly, having integrated twice	**DM1**
	Obtain exact value $\mathrm{e} - 2$	**A1**
	Use x- coordinate of Q found in part **(i)** and obtain final answer $\mathrm{e} - 2 + \dfrac{1}{\mathrm{e}}$	**B1√**
	Total:	**5**

187. 9709_s17_MS_31 Q: 3

	Answer	Mark
(i)	Remove logarithms correctly and obtain $e^x = \frac{1-y}{y}$	**B1**
	Obtain the **given answer** $y = \frac{e^{-x}}{1+e^{-x}}$ following full working	**B1**
	Total:	**2**
(ii)	State integral $k\ln(1+e^{-x})$ where $k = \pm 1$	***M1**
	State correct integral $-\ln(1+e^{-x})$	**A1**
	Use limits correctly	**DM1**
	Obtain the **given answer** $\ln\left(\frac{2e}{e+1}\right)$ following full working	**A1**
	Total:	**4**

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188. 9709_s17_MS_31 Q: 10

	Answer	Mark
(i)	State or imply $du = -\sin x\ dx$	**B1**
	Using correct double angle formula, express the integral in terms of u and du	**M1**
	Obtain integrand $\pm(2u^2-1)^2$	**A1**
	Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1}(2u^2-1)^2\,du$ with no errors seen	**A1**
	Substitute limits in an integral of the form au^5+bu^3+cu	**M1**
	Obtain answer $\frac{1}{15}(7-4\sqrt{2})$, or exact simplified equivalent	**A1**
	Total:	**6**

	Answer	Mark
(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**

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189. 9709_s17_MS_32 Q: 7

	Answer	Mark
(i)	Use quotient or chain rule	**M1**
	Obtain given answer correctly	**A1**
	Total:	**2**
(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and tanθ	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**
	OR2: Express LHS in terms of secθ and tanθ	**(M1**
	Multiply numerator and denominator by secθ + tanθ and use Pythagoras	**M1**
	Complete the proof	**A1)**
	Total:	**3**

	Answer	Mark
(iii)	Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$	B2
	Use correct limits correctly in an integral containing terms a tanθ and b secθ	M1
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	A1
	Total:	**4**

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190. 9709_s17_MS_33 Q: 4

	Answer	Mark
	Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta\ \mathrm{d}\theta$	*M1
	Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	A1
	Substitute limits correctly, having integrated twice	DM1
	Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	A1
	Total:	**4**

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191. 9709_s17_MS_33 Q: 7

	Answer	Mark
(i)	Use correct quotient rule or product rule	M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and solve for x	M1
	Obtain $x = 2$	A1
	Total:	**4**
(ii)	State or imply ordinates 1.6487…, 1.3591…, 1.4938…	B1
	Use correct formula, or equivalent, with $h = 1$ and three ordinates	M1
	Obtain answer 2.93 only	A1
	Total:	**3**
(iii)	Explain why the estimate would be less than E	B1
	Total:	**1**

192. 9709_s17_MS_33 Q: 9

	Answer	Mark
(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x+2}$	B1
	Use a relevant method to determine a constant	M1
	Obtain one of the values $A=3$, $B=-2$, $C=-6$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\frac{Ax+B}{x^2}+\frac{C}{3x+2}$ using same pattern of marks.]	A1
	Total:	**5**
(ii)	Integrate and obtain terms $3\ln x = \frac{2}{x} - 2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3\ln x + \frac{2}{x} - 3$ or equivalent	B3 FT
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x + \frac{b}{x} + c\ln(3x+2)$	M1
	Obtain the given answer following full and exact working	A1
	Total:	**5**

193. 9709_w17_MS_31 Q: 8

	Answer	Mark
(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

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194. 9709_w17_MS_31 Q: 9

	Answer	Mark
(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

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195. 9709_w17_MS_32 Q: 1

	Answer	Mark
(i)	State or imply ordinates 0.915929…, 1, 1.112485…	**B1**
	Use correct formula, or equivalent, with h = 1.2 and three ordinates	**M1**
	Obtain answer 2.42 only	**A1**
		3
(ii)	Justify the given statement	**B1**
		1

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196. 9709_m16_MS_32 Q: 5

	Answer	Mark	
(i)	State or imply $\mathrm{d}x = \sqrt{3}\sec^2\theta\,\mathrm{d}\theta$	**B1**	
	Substitute for x and $\mathrm{d}x$ throughout	**M1**	
	Obtain the given answer correctly	**A1**	[3]
(ii)	Replace integrand by $\frac{1}{2}\cos 2\theta + \frac{1}{2}$	**B1**	
	Obtain integral $\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$	**B1**√	
	Substitute limits correctly in an integral of the form $c\sin 2\theta + b\theta$, where $cb \neq 0$	**M1**	
	Obtain answer $\frac{1}{12}\sqrt{3}\pi + \frac{3}{8}$, or exact equivalent	**A1**	[4]
	[The f.t. is on integrands of the form $a\cos 2\theta + b$, where $ab \neq 0$.]		

197. 9709_m16_MS_32 Q: 9

	Answer	Mark	
(i)	State or obtain $A = 3$	**B1**	
	Use a relevant method to find a constant	**M1**	
	Obtain one of $B = -4$, $C = 4$ and $D = 0$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[5]
(ii)	Integrate and obtain $3x - 4\ln x$	**B1√**	
	Integrate and obtain term of the form $k\ln(x^2 + 2)$	**M1**	
	Obtain term $2\ln(x^2 + 2)$	**A1√**	
	Substitute limits in an integral of the form $ax + b\ln x + c\ln(x^2 + 2)$, where $abc \neq 0$	**M1**	
	Obtain given answer $3 - \ln 4$ after full and correct working	**A1**	[5]

198. 9709_s16_MS_31 Q: 2

Answer	Mark
Integrate by parts and reach $axe^{-2x} + b\int e^{-2x}\,dx$	**M1**
Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$, or equivalent	**A1**
Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent	**A1**
Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice	**M1**
Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent	**A1**
	[5]

199. 9709_s16_MS_32 Q: 3

Answer	Mark	
Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x\,dx$	**M1***	
Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent	**A1**	
Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent	**A1**	
Use limits correctly having integrated twice	**DM1***	
Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen	**A1**	[5]

200. 9709_s16_MS_32 Q: 7

	Answer	Mark	
(i)	State or imply the form $A+\dfrac{B}{2x+1}+\dfrac{C}{x+2}$	**B1**	
	State or obtain $A = 2$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $B = 1$, $C = -2$	**A1**	
	Obtain the other value	**A1**	[5]
(ii)	Integrate and obtain terms $2x+\frac{1}{2}\ln(2x+1)-2\ln(x+2)$	**B3√**	
	Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$	**M1**	
	Obtain the given answer after full and correct working	**A1**	[5]

201. 9709_s16_MS_33 Q: 7

	Answer	Mark
(i)	State or imply $\mathrm{d}u = 2x\,\mathrm{d}x$, or equivalent	**B1**
	Substitute for x and $\mathrm{d}x$ throughout	**M1**
	Reduce to the given form and justify the change in limits	**A1**
		[3]
(ii)	Convert integrand to a sum of integrable terms and attempt integration	**M1**
	Obtain integral $\frac{1}{2}\ln u+\dfrac{1}{u}-\dfrac{1}{4u^2}$, or equivalent	**A1 + A1**
	(deduct A1 for each error or omission)	
	Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2}	**M1**
	Obtain answer $\frac{1}{2}\ln 2-\frac{5}{16}$, exact simplified equivalent	**A1**
		[5]

202. 9709_w16_MS_31 Q: 5

	Answer	Mark	
(i)	*EITHER*: Use tan 2*A* formula to express LHS in terms of $\tan\theta$	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos 2*A* formula	**M1**	
	Obtain the given result correctly	**A1**	
	OR: Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos 2*A* formula or sin(*A* – *B*) formula	**M1**	
	Obtain the given result correctly	**A1**	[4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	**M1***	
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta)+\ln(\cos\theta)$, or equivalent	**A1**	
	Substitute limits correctly (expect to see use of both limits)	**DM1**	
	Obtain the given answer following full and correct working	**A1**	[4]

203. 9709_w16_MS_31 Q: 7

	Answer	Mark	
(i)	Use the correct product rule	**M1**	
	Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x}+\frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x=\sqrt{5}-1$ only	**A1**	[4]
(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x}+b\int(2-2x)e^{\frac{1}{2}x}\,dx$	**M1***	
	Obtain $2e^{\frac{1}{2}x}(2x-x^2)-2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent	**A1**	
	Complete the integration correctly, obtaining $(12x-2x^2-24)e^{\frac{1}{2}x}$, or equivalent	**A1**	
	Use limits $x=0$, $x=2$ correctly having integrated by parts twice	**DM1**	
	Obtain answer 24 – 8e, or <u>exact</u> simplified equivalent	**A1**	[5]

204. 9709_w16_MS_33 Q: 6

	Answer	Mark	
(i)	State or imply $du=\dfrac{1}{2\sqrt{x}}dx$	**B1**	
	Substitute for x and dx throughout	**M1**	
	Justify the change in limits and obtain the given answer	**A1**	[3]
(ii)	Convert integrand into the form $A+\dfrac{B}{u+1}$	**M1***	
	Obtain integrand $A=1$, $B=-2$	**A1**	
	Integrate and obtain $u-2\ln(u+1)$	**A1✓ + A1✓**	
	Substitute limits correctly in an integral containing terms au and $b\ln(u+1)$, where $ab\neq 0$	**DM1**	
	Obtain the given answer following full and correct working [The f.t. is on A and B.]	**A1**	[6]

205. 9709_s15_MS_31 Q: 2

Answer	Mark	
Attempt calculation of at least 3 ordinates	M1	
Obtain 9, 7, 1, 17	A1	
Use trapezium rule with $h=1$	M1	
Obtain $\frac{1}{2}(9+14+2+17)$ or equivalent and hence 21	A1	[4]

206. 9709_s15_MS_31 Q: 5

	Answer	Mark	
(a)	Use identity $\tan^2 2x = \sec^2 2x - 1$	B1	
	Obtain integral of form $ax + b\tan 2x$	M1	
	Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of $+\,c$	A1	[3]
(b)	State $\sin x \cos\frac{1}{2}\pi + \cos x\ \sin\frac{1}{6}\pi$	B1	
	Simplify integrand to $\cos\frac{1}{6}\pi + \frac{\cos x \sin\frac{1}{6}\pi}{\sin x}$ or equivalent	B1	
	Integrate to obtain at least term of form a In(sin x)	*M1	
	Apply limits and simplify to obtain two terms	M1 dep *M	
	Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$ or equivalent	A1	[5]

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207. 9709_s15_MS_31 Q: 9

	Answer	Mark	
(i)	Use product rule to find first derivative	M1	
	Obtain $2xe^{2-x} - x^2e^{2-x}$	A1	
	Confirm $x = 2$ at M	A1	[3]
(ii)	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm \int 2xe^{2-x}\,dx$	*M1	
	Obtain $-x^2e^{2-x} + \int 2xe^{2-x}\,dx$	A1	
	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm 2xe^{2-x} \pm 2e^{2-x}$	*M1	
	Obtain $-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$	A1	
	Use limits 0 and 2 having integrated twice	M1 dep *M	
	Obtain $2e^2 - 10$	A1	[6]

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208. 9709_s15_MS_32 Q: 1

Answer	Mark	
State or imply ordinates 0, 0.405465…, 0.623810…, 0.693147…	B1	
Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
Obtain answer 0.72	A1	**[3]**

209. 9709_s15_MS_32 Q: 6

	Answer	Mark	
(i)	State or imply $\mathrm{d}u = -\frac{1}{2\sqrt{x}}\mathrm{d}x$, or equivalent	B1	
	Substitute for x and $\mathrm{d}x$ throughout	M1	
	Obtain integrand $\frac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
	Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
(ii)	Integrate and obtain at least two terms of the form $a\ln u, bu,$ and cu^2	M1*	
	Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
	Substitute limits correctly	M1(dep*)	
	Obtain the given answer correctly having shown sufficient working	A1	**[4]**

210. 9709_s15_MS_33 Q: 10

	Answer	Mark	
(i)	State or imply $\mathrm{f}(x) \equiv \frac{A}{2x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A = 2$, $B = -1$, $C = 3$	A1	
	Obtain the remaining values A1 +	A1	**5**
	[Apply an analogous scheme to the form $\frac{A}{2x-1} + \frac{Dx+E}{(x+2)^2}$; the values being $A = 2$, $D = -1$, $E = 1$.]		
(ii)	Integrate and obtain terms $\frac{1}{2} \cdot 2\ln(2x-1) - \ln(x+2) - \frac{3}{x+2}$	B1√ + B1√ + B1√	
	Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG.	M1	
	Obtain the given answer following full and exact working	A1	**5**
	[The t marks are dependent on A, B, C etc.]		
	[SR: If B, C or E omitted, give B1M1 in part (**i**) and B1√B1√M1 in part (**ii**).]		
	[NB: Candidates who follow the A, D, E scheme in part (**i**) and then integrate $\frac{-x+1}{(x+2)^2}$ by parts should obtain $\frac{1}{2} \cdot 2\ln(2x-1) - \ln(x+2) + \frac{x-1}{x+2}$ (the third term is equivalent to $-\frac{3}{x+2}+1$).]		

211. 9709_w15_MS_31 Q: 10

	Answer	Mark	
(i)	Use the quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	**A1**	[4]
(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	**M1**	
	State indefinite integral $\frac{1}{3}\ln(1+x^3)$	**A1**	
	Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	**M1**	
	State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3)-\frac{1}{3}\ln 2$, or equivalent	**A1**	
	Use a correct method for finding p from an equation of the form $\ln(1+p^3)=a$ or $\ln((1+p^3)/2)=b$	**M1**	
	Obtain answer $p = 3.40$	**A1**	[2]

212. 9709_w15_MS_33 Q: 5

Answer	Mark	
State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent	**B1**	
Use identity $\sin 2x = 2\sin x\cos x$	**B1**	
Carry out complete substitution, for x and $\mathrm{d}x$	**M1**	
Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent	**A1**	
Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$	**M1***	
Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
Apply correct limits correctly	**dep M1***	
Obtain $\frac{20}{3}$ or exact equivalent	**A1**	[8]

213. 9709_w15_MS_33 Q: 7

		Answer	Mark	
(i)	Either	Substitute $x=-1$ and evaluate	**M1**	
		Obtain 0 and conclude $x+1$ is a factor	**A1**	
	Or	Divide by $x+1$ and obtain a constant remainder	**M1**	
		Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]
(ii)	Attempt division, or equivalent, at least as far as quotient $4x^2+kx$		**M1**	
	Obtain complete quotient $4x^2-5x-6$		**A1**	
	State form $\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{4x+3}$		**A1**	
	Use relevant method for finding at least one constant		**M1**	
	Obtain one of $A=-2, B=1, C=8$		**A1**	
	Obtain all three values		**A1**	
	Integrate to obtain three terms each involving natural logarithm of linear form		**M1**	
	Obtain $-2\ln(x+1)+\ln(x-2)+2\ln(4x+3)$, condoning no use of modulus signs and absence of … $+c$		**A1**	[8]

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214. 9709_s20_MS_31 Q: 6

(a)	State or imply $AT=r\tan x$ or $BT=r\tan x$	**B1**
	Use correct area formula and form an equation in r and x	**M1**
	Rearrange in the given form	**A1**
		3
(b)	Calculate the values of a relevant expression or pair of expressions at $x=1$ and $x=1.4$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.35	**A1**
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	**A1**
		3

215. 9709_s20_MS_32 Q: 9

(a)	State $\cos p = \frac{k}{1+p}$	**B1**
	Differentiate both equations and equate derivatives at $x = p$	**M1**
	Obtain a correct equation in any form, e.g. $-\sin p = -\frac{k}{(1+p)^2}$	**A1**
	Eliminate k	**M1**
	Obtain the **given answer** showing sufficient working	**A1**
		5
(b)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer $p = 0.568$	**A1**
	Show sufficient iterations to justify 0.568 to 3 d.p., or show there is a sign change in the interval (0.5675, 0.5685)	**A1**
		3
(c)	Use a correct method to find k	**M1**
	Obtain answer $k = 1.32$	**A1**
		2

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216. 9709_s20_MS_33 Q: 6

(a)	Sketch a relevant graph, e.g. $y = x^5$	**B1**
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	**B1**
		2
(b)	State a suitable equation, e.g. $x = \frac{4x^5 + 2}{5x^4 - 1}$	**B1**
	Rearrange this as $x^5 = 2 + x$ or commence working *vice versa*	**B1**
		2
(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.267	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	**A1**
		3

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217. 9709_w20_MS_31 Q: 5

	Answer	Mark	Partial Marks
(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec x, U shaped, roughly symmetrical about $x=\frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right)=1$ and domain at least $\left(\frac{\pi}{6},\frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y=1+e^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

218. 9709_w20_MS_32 Q: 10

	Answer	Mark	Partial Marks
(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{x}}\cos x-\sqrt{x}\sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a=\frac{1}{2a}$	**A1**	Obtain given answer from correct working. The question says 'show that ..' so there should be an intermediate step e.g. $\cos x=2x\sin x$. Allow $\tan x=\frac{1}{2x}$
		3	
(b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	**M1**	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	**A1**	Clear conclusion
	Show sufficient iterations to at least 4 d.p.to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	**A1**	3, 3.3067, 3.2917, 3.2923 Allow more than 4d.p. Condone truncation.
		3	
(c)	State or imply the indefinite integral for the volume is $\pi\int\left(\sqrt{x}\cos x\right)^2\mathrm{d}x$	**B1**	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct cos 2*A* formula, commence integration by parts and reach $x(ax+b\sin 2x)\pm\int ax+b\sin 2x\mathrm{d}x$	***M1**	Alternative: $\frac{x^2}{4}+\frac{x}{4}\sin 2x-\int\frac{1}{4}\sin 2x\mathrm{d}x$
	Obtain $x(\frac{1}{2}x+\frac{1}{4}\sin 2x)-\int\frac{1}{2}x+\frac{1}{4}\sin 2x\,\mathrm{d}x$, or equivalent	**A1**	
	Complete integration and obtain $\frac{1}{4}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x$	**A1**	OE
	Substitute limits $x=0$ and $x=\frac{1}{2}\pi$, having integrated twice	**DM1**	$\frac{\pi}{2}\left[\frac{\pi^2}{8}+0-\frac{1}{4}-0-0-\frac{1}{4}\right]$
	Obtain answer $\frac{1}{16}\pi\left(\pi^2-4\right)$, or exact equivalent	**A1**	CAO
		6	

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219. 9709_m19_MS_32 Q: 2

	Answer	Mark	Partial Marks
(i)	Use the iterative formula correctly at least once	**M1**	
	Obtain answer 1.3195	**A1**	
	Show sufficient iterations to 6 d.p. to justify 1.3195 to 4 d.p., or show there is a sign change in (1.31945, 1.31955)	**A1**	
		3	
(ii)	State $x=\frac{2x^6+12x}{3x^5+8}$, or equivalent	**B1**	
	State answer $\sqrt[5]{4}$, or exact equivalent	**B1**	
		2	

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220. 9709_s19_MS_31 Q: 7

	Answer	Mark	Partial Marks
(i)	State at least one correct derivative	**B1**	$-2\sin\frac{1}{2}x$, $\frac{1}{(4-x)^2}$
	Equate product of derivatives to – 1	**M1**	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x=(4-x)^2$	**A1**	
	Rearrange correctly to obtain $a=4-\sqrt{2\sin\frac{a}{2}}$ **AG**	**A1**	
		4	
(ii)	Calculate values of a relevant expression or pair of expressions at $a=2$ and $a=3$	**M1**	e.g. $\begin{matrix} a=2 & 2<2.7027.. \\ a=3 & 3>2.587.. \end{matrix}$ $\begin{pmatrix} 0.703 \\ -0.412 \end{pmatrix}$ $\begin{pmatrix} 2.317 \\ -0.995 \end{pmatrix}$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	**A1**	
		2	
(iii)	Use the iterative formula $a_{n+1}=4-\sqrt{(2\sin\frac{1}{2}a_n)}$ correctly at least once	**M1**	
	Obtain final answer 2.611	**A1**	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	**A1**	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
		3	

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221. 9709_s19_MS_32 Q: 6

	Answer	Mark	Partial Marks
(i)	Correct use of trigonometry to obtain $AB = 2r\cos x$	B1	AG
		1	
(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	M1	$\frac{1}{2}\times\frac{1}{2}\pi r^2 = \frac{1}{2}(2r\cos x)^2\,2x$
	Obtain $x = \cos^{-1}\sqrt{\frac{\pi}{16x}}$ correctly **AG**	A1	Via correct simplification e.g. from $\cos^2 x = \frac{\pi}{16x}$
		2	
(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ Must be working in radians	M1	e.g. $\begin{matrix} x=1 & 1\to 1.11 \\ x=1.5 & 1.5\to 1.20 \end{matrix}$ Accept $\begin{matrix} \mathrm{f}(1)=1.11 \\ \mathrm{f}(1.5)=1.20 \end{matrix}$ $\mathrm{f}(x) = x-\cos^{-1}\sqrt{\frac{\pi}{16x}}$ $:\mathrm{f}(1) = -0.111.,\ \mathrm{f}(1.5) = 0.3..$ $\mathrm{f}(x) = \cos x-\sqrt{\frac{\pi}{16x}}$ $:\mathrm{f}(1) = 0.097.,\ \mathrm{f}(1.5) = -0.291.$ For $16x\cos^2 x-\pi$ $\mathrm{f}(1) = 1.529..,\ \mathrm{f}(1.5) = -3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	A1	
		2	
(iv)	Use $x_{n+1} = \cos^{-1}\sqrt{\left(\frac{\pi}{16x_n}\right)}$ correctly at least twice Must be working in radians	M1	$1, 1.11173, 1.13707, 1.14225, 1.14329, 1.14349,$ $1.14354, 1.14354$ $1.25, 1.16328, 1.14742, 1.14432, 1.14370$ $1.5, 1.20060, 1.15447, 1.14570, 1.14397, 1.14363$
	Obtain final answer 1.144	A1	
	Show sufficient iterations to at least 5 d.p. to justify 1.144 to 3 d.p. or show there is a sign change in the interval (1.1435, 1.1445)	A1	
		3	

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222. 9709_s19_MS_33 Q: 6

	Answer	Mark	Partial Marks
(i)	State $b = 3$	B1	
		1	
(ii)	Commence division by $x-b$ and reach partial quotient x^3+kx^2	M1	
	Obtain quotient x^3+x^2+3x+2	A1	There being no remainder
	Equate quotient to zero and rearrange to make the subject a	M1	
	Obtain the given equation	A1	
		4	
(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}\left(2+a_n{}^2+a_n{}^3\right)$ correctly at least once	M1	
	Obtain final answer –0.715	A1	
	Show sufficient iterations to 5 d.p. to justify –0.715 to 3 d.p., or show there is a sign change in the interval (–0.7145, –0.7155)	A1	
		3	

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223. 9709_w19_MS_31 Q: 5

	Answer	Mark	Partial Marks
(i)	Use correct product rule	M1	
	Obtain correct derivative in any form $\frac{dy}{dx}=-2e^{-2x}\ln(x-1)+\frac{e^{-2x}}{x-1}$	A1	
	Equate derivative to zero and derive $x=1+e^{\frac{1}{2(x-1)}}$ or $p=1+^{\frac{1}{2(p-1)}}$	A1	AG
		3	
(ii)	Calculate values of a relevant expression or pair of relevant expressions at x = 2.2 and x = 2.6 $f(x)=\ln(x-1)-\frac{1}{2(x-1)}\Rightarrow f(2.2)=-0.234, f(2.6)=0.317$ $f(x)=2e^{-2x}\ln(x-1)+\frac{e^{-2x}}{x-1}\Rightarrow f(2.2)=0.005\ldots, f(2.6)=-0.0017\ldots$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
(iii)	Use the iterative process $p_{n+1}=1+\exp\left(\frac{1}{2(p_n-1)}\right)$ correctly at least once	M1	
	Obtain final answer 2.42	A1	
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval (2.415, 2.425)	A1	
		3	

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224. 9709_w19_MS_32 Q: 9

	Answer	Mark	Partial Marks
(i)	Commence integration by parts, reaching $ax\sin\frac{1}{3}x - b\int\sin\frac{1}{3}x\,dx$	*M1	
	Obtain $3x\sin\frac{1}{3}x - 3\int\sin\frac{1}{3}x\,dx$	A1	
	Complete integration and obtain $3x\sin\frac{1}{3}x + 9\cos\frac{1}{3}x$	A1	
	Substitute limits correctly and equate result to 3 in an integral of the form $px\sin\frac{1}{3}x + q\cos\frac{1}{3}x$	DM1	$3 = 3a\sin\frac{a}{3} + 9\cos\frac{a}{3}(-0) - 9$
	Obtain $a = \dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ correctly	A1	With sufficient evidence to show how they reach the given equation
		5	
(ii)	Calculate values at $a = 2.5$ and $a = 3$ of a relevant expression or pair of expressions.	M1	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving f(a) = 0, f(2.5) = 0.179. and f(3) = –0.173 or if $f(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4 \Rightarrow f(2.5) = -0.13.., f(3) = 0.145...$
	Complete the argument correctly with correct calculated values	A1	Accept values to 1 sf. or better
		2	
(iii)	Use the iterative process $a_{n+1} = a_{n+1}\dfrac{4 - 3\cos\frac{1}{3}a_n}{\sin\frac{1}{3}a_n}$ correctly at least once	M1	
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	A1	
	Obtain final answer 2.736	A1	
		3	

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225. 9709_w19_MS_33 Q: 5

	Answer	Mark	Partial Marks
(i)	Sketch a relevant graph, e.g. $y = \ln(x + 2)$	B1	
	Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement	B1	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
(ii)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
(iii)	Use the iterative formula correctly at least **twice** using output from a previous iteration	M1	
	Obtain final answer 1.23	A1	
	Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)	A1	
		3	

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226. 9709_m18_MS_32 Q: 7

	Answer	Mark
(i)	Sketch a relevant graph, e.g. $y = e^{2x}$	B1
	Sketch a second relevant graph, e.g. $y = 6 + e^{-x}$, and justify the given statement	B1
		2
(ii)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 0.5$ and $x = 1$	M1
	Complete the argument correctly with correct calculated values	A1
		2
(iii)	State a suitable equation, e.g. $x = \frac{1}{3}\ln\left(1 + 6e^{x}\right)$	B1
	Rearrange this as $e^{2x} = 6 + e^{-x}$, or commence working *vice versa*	B1
		2

	Answer	Mark
(iv)	Use the iterative formula correctly at least once	M1
	Obtain final answer 0.928	A1
	Show sufficient iterations to 5 d.p. to justify 0.928 to 3 d.p., or show there is a sign change in the interval (0.9275, 0.9285)	A1
		3

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227. 9709_s18_MS_31 Q: 8

(i)	Integrate by parts and reach $lxe^{-\frac{1}{2}x} + m\int e^{-\frac{1}{2}x}\ \mathrm{d}x$	**M1***
	Obtain $-2xe^{-\frac{1}{2}x} + 2\int e^{-\frac{1}{2}x}\ \mathrm{d}x$	**A1**
	Complete the integration and obtain $-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Having integrated twice, use limits and equate result to 2	**M1(dep*)**
	Obtain the given equation correctly	**A1**
		5
(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(iii)	Use the iterative formula $a_{n+1} = 2\ln(a_n + 2)$ correctly at least once	**M1**
	Obtain final answer 3.36	**A1**
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	**A1**
		3

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228. 9709_s18_MS_32 Q: 6

	Answer	Mark	Partial Marks
(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	**M1**	e.g. $\frac{\pi a^2}{4} = \frac{1}{2}a^2\theta - \frac{1}{2}a^2\sin\theta$
	State a correct equation in any form	**A1**	Given answer so check working carefully
	Obtain the given answer correctly	**A1**	
		3	
(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	**M1**	e.g. $\mathrm{f}(\theta) = \frac{\pi}{2} + \sin\theta \quad \begin{cases} \mathrm{f}(2.2) = 2.37... > 2.2 \\ \mathrm{f}(2.4) = 2.24... < 2.4 \end{cases}$ or $\mathrm{f}(\theta) = \theta - \frac{\pi}{2} - \sin\theta \quad \begin{cases} \mathrm{f}(2.2) = -0.17... < 0 \\ \mathrm{f}(2.4) = +0.15... > 0 \end{cases}$
	Complete the argument correctly with correct calculated values	**A1**	
		2	

	Answer	Mark	Partial Marks
(iii)	Use $\theta_{n+1} = \frac{1}{2}\pi + \sin\theta_n$ correctly at least once	M1	e.g.
	Obtain final answer 2.31	A1	
	Show sufficient iterations to 4 d.p. to justify 2.31 to 2 d.p. or show there is a sign change in the interval (2.305, 2.315)	A1	
		3	

2.2	2.3	2.4
2.3793	2.3165	2.2463
2.2614	2.3054	2.3512
2.3417	2.3129	2.2814
2.2881	2.3079	2.3288
2.3244		2.2970
2.3000		2.3185
2.3165		2.3041
2.3054		2.3138
2.3129		2.3072

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229. 9709_s18_MS_33 Q: 4

	Answer	Mark
(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain the given equation	**A1**
	Total:	**3**
(ii)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \frac{3}{x}$, and justify the given statement	**B1**
	Total:	**2**
(iii)	Use iterative formula $x_{n+1} = \frac{3 + x}{\ln x_n}$ correctly at least once	**M1**
	Obtain final answer 4.97	**A1**
	Show sufficient iterations to 4 d.p.to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	**A1**
	Total:	**3**

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230. 9709_w18_MS_31 Q: 3

	Answer	Mark	Partial Marks
(i)	Sketch a relevant graph, e.g. $y = x^3$	B1	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	B1	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	B1	
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	B1	
		2	

	Answer	Mark	Partial Marks
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.213	A1	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	A1	
		3	

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231. 9709_w18_MS_32 Q: 5

	Answer	Mark	Partial Marks
(i)	Use product rule on a correct expression	M1	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	A1	$\frac{\mathrm{d}y}{\mathrm{d}x} = \ln\left(8-x\right) - \frac{x}{8-x}$
	Equate derivative to 1 and obtain $x = 8 - \frac{8}{\ln\left(8-x\right)}$	A1	**Given answer:** check carefully that it follows from correct working
			Condone the use of *a* for *x* throughout
		3	
(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.9$ and $x = 3.1$	M1	$8 - \frac{8}{\ln 5.1} = 3.09 > 2.9, \quad 8 - \frac{8}{\ln 4.9} = 2.97 < 3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	A1	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

	Answer	Mark	Partial Marks
(iii)	Use the iterative process $x_{n+1} = 8 - \frac{8}{\ln\left(8-x_n\right)}$ correctly to find at least two successive values. SR: Clear successive use of 0, 1, 2, 3 etc., or equivalent, scores M0.	M1	3, 3.0293, 3.0111, 3.0225, 3.0154, (3.0198) 2.9, 3.0897, 2.9728, 3.0460, 3.0006, 3.290, 3.0113, 3.0223, 3.0155 3.1, 2.9661, 3.0501, 2.9980, 3.0305, 3.0103, 3.0229, 3.0151 Allow M1 if values given to fewer than 4 dp
	Obtain final answer 3.02	A1	
	Show sufficient iterations to at least 4 d.p. to justify 3.02 to 2 d.p., or show there is a sign change in the interval (3.015, 3.025)	A1	Must have two consecutive values rounding correctly to 3.02
		3	

232. 9709_s17_MS_31 Q: 5

	Answer	Mark
(i)	Use correct sector formula at least once and form an equation in r and x	**M1**
	Obtain a correct equation in any form	**A1**
	Rearrange in the given form	**A1**
	Total:	**3**
(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.374	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval (1.3745, 1.3755)	**A1**
	Total:	**3**

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233. 9709_s17_MS_32 Q: 10

	Answer	Mark
(i)	Use correct product rule	**M1**
	Obtain correct derivative in any form $\left(y' = 2x\cos 2x - 2x^2 \sin 2x\right)$	**A1**
	Equate to zero and derive the given equation	**A1**
	Total:	**3**
(ii)	Use the iterative formula correctly at least once e.g. $0.5 \to 0.55357 \to 0.53261 \to 0.54070 \to 0.53755$	**M1**
	Obtain final answer 0.54	**A1**
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	**A1**
	Total:	**3**

	Answer	Mark
(iii)	Integrate by parts and reach $ax^2 \sin 2x + b\int x \sin 2x \, \mathrm{d}x$	*M1
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x.\frac{1}{2}\sin 2x \, \mathrm{d}x$	A1
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	A1
	Substitute limits $x = 0,\ x = \frac{1}{4}\pi$, having integrated twice	DM1
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	A1
	Total:	5

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234. 9709_s17_MS_33 Q: 6

	Answer	Mark
(i)	Calculate the value of a relevant expression or expressions at $x = 2.5$ and at another relevant value, e.g. $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	2
(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}(1/(1-x))$ without suffices	B1
	Rearrange this as $\cot x = 1 - x$, or commence working *vice versa*	B1
	Total:	2
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 2.576 only	A1
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	A1
	Total:	3

235. 9709_w17_MS_31 Q: 3

	Answer	Mark
(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(ii)	Use an iterative formula correctly at least once	**M1**
	Show that (B) fails to converge	**A1**
	Using (A), obtain final answer 2.43	**A1**
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	**A1**
		4

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236. 9709_w17_MS_32 Q: 9

	Answer	Mark
(i)	Integrate by parts and reach $ax^{\frac{3}{2}}\ln x + b\int x^{\frac{3}{2}}.\frac{1}{x}\,\mathrm{d}x$	***M1**
	Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,\mathrm{d}x$	**A1**
	Obtain integral $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	**A1**
	Substitute limits correctly and equate to 2	**DM1**
	Obtain the given answer correctly AG	**A1**
		5
(ii)	Evaluate a relevant expression or pair of expressions at $x = 2$ and $x = 4$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 3.031	**A1**
	Show sufficient iterations to 5 d.p. to justify 3.031 to 3 d.p., or show there is a sign change in the interval (3.0305, 3.0315)	**A1**
		3

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237. 9709_m16_MS_32 Q: 3

	Answer	Mark	
(i)	Consider sign of $x^5 - 3x^3 + x^2 - 4$ at $x = 1$ and $x = 2$, or equivalent	M1	
	Complete the argument correctly with correct calculated values	A1	[2]
(ii)	Rearrange the given quintic equation in the given form, or work *vice versa*	B1	[1]
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.78	A1	
	Show sufficient iterations to 4 d.p. to justify 1.78 to 2 d.p., or show there is a sign change in the interval (1.775, 1.785)	A1	[3]

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238. 9709_s16_MS_31 Q: 6

	Answer	Mark
(i)	Make recognizable sketch of a relevant graph	B1
	Sketch the other relevant graph and justify the given statement	B1 [2]
(ii)	State $x = \frac{1}{2}\ln(25/x)$	B1
	Rearrange this in the form $5e^{-x} = \sqrt{x}$	B1 [2]
(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 1.43	A1
	Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the interval (1.425, 1.435)	A1 [3]

239. 9709_s16_MS_32 Q: 8

	Answer	Mark	
(i)	Use correct quotient or chain rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Obtain the given answer correctly	**A1**	[3]
(ii)	State a correct equation, e.g. $-e^{-a} = -\operatorname{cosec} a \cot a$	**B1**	
	Rearrange it correctly in the given form	**B1**	[2]
(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	[2]
(iv)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.317	**A1**	
	Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign change in the interval (1.3165, 1,3175)	**A1**	[3]

240. 9709_s16_MS_33 Q: 6

	Answer	Mark
(i)	Use the product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate 2-term derivative to zero and obtain the given answer correctly	**A1**
		[3]
(ii)	Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or compare values of relevant expressions at $p = 2$ and $p = 2.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		[2]
(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 2.15	**A1**
	Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change in the interval (2.145,2.155)	**A1**
		[3]

241. 9709_w16_MS_31 Q: 6

	Answer	Mark	
(i)	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	**B1** **B1**	[2]
(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	**M1** **A1**	[2]
(iii)	State $x = 2\sin^{-1}\left(\frac{3}{x+3}\right)$ Rearrange this in the form $\operatorname{cosec}\frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin\frac{x}{2} = \left(\frac{3}{x+3}\right)$ for first B1	**B1** **B1**	[2]
(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**M1** **A1** **A1**	[3]

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242. 9709_w16_MS_33 Q: 9

	Answer	Mark	
(i)	Differentiate both equations and equate derivatives Obtain equation $\cos a - a\sin a = -\frac{k}{a^2}$ State $a\cos a = \frac{k}{a}$ and eliminate k Obtain the given answer showing sufficient working	**M1*** **A1 + A1** **DM1** **A1**	[5]
(ii)	Show clearly correct use of the iterative formula at least once Obtain answer 1.077 Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)	**M1** **A1** **A1**	[3]
(iii)	Use a correct method to determine k Obtain answer $k = 0.55$	**M1** **A1**	[2]

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243. 9709_s15_MS_31 Q: 10

		Answer	Mark	
(i)		Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain $\frac{dy}{dx} = \frac{1}{2}(3t^2 + 2)(t + 2)$	A1	
		Identify value of t at the origin as –1	B1	
		Substitute to obtain $\frac{5}{2}$ as gradient at the origin	A1	[5]
(ii)	**(a)**	Equate derivative to $\frac{1}{2}$ and confirm $p = \frac{1}{3p^2 + 2} - 2$	B1	[1]
	(b)	Use the iterative formula correctly at least once	M1	
		Obtain value $p = -1.924$ or better (–1.92367…)	A1	
		Show sufficient iterations to justify accuracy or show a sign change in appropriate interval	A1	
		Obtain coordinates (–5.15, –7.97)	A1	[4]

244. 9709_s15_MS_32 Q: 5

	Answer	Mark	
(i)	State or imply $AT = r\tan x$ or $BT = r\tan x$	B1	
	Use correct arc formula and form an equation in r and x	M1	
	Rearrange in the given form	A1	**[3]**
(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.3$	M1	
	Complete the argument correctly with correct calculated values	A1	**[2]**
(iii)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 1.11	A1	
	Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a sign change in the interval (1.105, 1.115)	A1	**[3]**

245. 9709_s15_MS_33 Q: 6

	Answer		Mark	
(i)	Integrate and reach $\pm x\sin x \mp \int \sin x \, dx$		M1*	
	Obtain integral $x\sin x + \cos x$		A1	
	Substitute limits correctly, must be seen since AG, and equate result to 0.5		M1(dep*)	
	Obtain the given form of the equation		A1	**4**
(ii)	*EITHER*:	Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leqslant \frac{\pi}{2}$	M1	
	OR:	Using limits correctly, consider the sign of $[x\sin x + \cos x]_0^a - 0.5$, or compare the value of $[x\sin x + \cos x]_0^a$ with 0.5, for a =1 AND for another relevant value, e.g $a = 1.5 \leqslant \frac{\pi}{2}$.	M1	
	Complete the argument, so change of sign, or above and below stated, both with correct calculated values		A1	**2**
(iii)	Use the iterative formula correctly at least once		M1	
	Obtain final answer 1.2461		A1	
	Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615)		A1	**3**

246. 9709_w15_MS_31 Q: 4

	Answer	Mark	
(i)	Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x, or equivalent	**M1**	
	Obtain the pair $x = 2$ and $x = 3$, with no errors seen	**A1**	[2]
(ii)	State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$	**B1**	
	Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa*	**B1**	[2]
(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 2.219	**A1**	
	Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195)	**A1**	[3]

247. 9709_w15_MS_33 Q: 4

	Answer	Mark	
(i)	Use $\dfrac{dy}{dx}=\dfrac{\dot{y}}{\dot{x}}$ and equate $\dfrac{dy}{dx}$ to 4	**M1**	
	Obtain $\dfrac{4p^3}{2p+3}=4$ or equivalent	**A1**	
	Confirm given result $p=\sqrt[3]{2p+3}$ correctly	**A1**	[3]
(ii)	Evaluate $p-\sqrt[3]{2p+3}$ or p^3-2p-3 or equivalent at 1.8 and 2.0	**M1**	
	Justify result with correct calculations and argument (–0.076 and 0.087 or –0.77 and 1 respectively)	**A1**	[2]
(iii)	Use the iterative process correctly at least once with $1.8 \leqslant p_n \leqslant 2.0$	**M1**	
	Obtain final answer 1.89	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895)	**A1**	[3]

248. 9709_s20_MS_31 Q: 9

(a)	State $\overrightarrow{AB}\left(\text{or } \overrightarrow{BA}\right)$ and $\overrightarrow{BC}\left(\text{or } \overrightarrow{CB}\right)$ in vector form	**B1**
	Calculate their scalar product	**M1**
	Show product is zero and confirm angle *ABC* is a right angle	**A1**
		3
(b)	Use correct method to calculate the lengths of *AB* and *BC*	**M1**
	Show that $AB = BC$ and the triangle is isosceles	**A1**
		2
(c)	State a correct equation for the line through *B* and *C*, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(2\mathbf{i}+\mathbf{j}+2\mathbf{k})$ or $\mathbf{r}=3\mathbf{i}+2\mathbf{j}+3\mathbf{k}+\mu(-2\mathbf{i}-\mathbf{j}-2\mathbf{k})$	**B1**
	Taking a general point of *BC* to be *P*, form an equation in λ by **either** equating the scalar product of $\overrightarrow{OP}$ and $\overrightarrow{BC}$ to zero, **or** applying Pythagoras to triangle *OBP* (or *OCP*), *or* setting the derivative of $\left\lvert\overrightarrow{OP}\right\rvert$ to zero	**M1**
	Solve and obtain $\lambda=-\dfrac{5}{9}$	**A1**
	Obtain answer $\dfrac{1}{3}\sqrt{2}$, or equivalent	**A1**
	Alternative method for question 9(c)	
	Use a scalar product to find the projection *CN* (or *BN*) of *OC* (or *OB*) on *BC*	**M1**
	Obtain answer $CN=\dfrac{5}{3}\left(\text{or } BN=\dfrac{14}{3}\right)$	**A1**
	Use Pythagoras to find *ON*	**M1**
	Obtain answer $\dfrac{1}{3}\sqrt{2}$, or equivalent	**A1**
		4

249. 9709_s20_MS_32 Q: 10

(a)	State that the position vector of M is $3\mathbf{i}+\mathbf{j}$	**B1**
	Use a correct method to find the position vector of N	**M1**
	Obtain answer $\frac{10}{3}\mathbf{i}+2\mathbf{j}+2\mathbf{k}$	**A1**
	Use a correct method to form an equation for MN	**M1**
	Obtain correct answer in any form, e.g. $\mathbf{r}=3\mathbf{i}+\mathbf{j}+\lambda\left(\frac{1}{3}\mathbf{i}+\mathbf{j}+2\mathbf{k}\right)$	**A1**
		5
(b)	State or imply $\mathbf{r}=\mu(2\mathbf{i}+2\mathbf{j}+3\mathbf{k})$ as equation for OB	**B1**
	Equate sufficient components of MN and OB and solve for λ or for μ	**M1**
	Obtain $\lambda=3$ or $\mu=2$ and position vector $4\mathbf{i}+4\mathbf{j}+6\mathbf{k}$ for P	**A1**
		3
(c)	Carry out correct process for evaluating the scalar product of direction vectors for OP and MP, or equivalent	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**
	Obtain answer 21.6°	**A1**
		3

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250. 9709_s20_MS_33 Q: 8

(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of C	**M1**
	Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**
	Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and AD	**M1**
	Show that $ABCD$ has a pair of unequal adjacent sides	**A1**
	Alternative method for question 8(a)	
	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of C	**M1**
	Obtain answer $4\mathbf{i}+3\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**
	Use the correct process to calculate the scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$, or equivalent	**M1**
	Show that the diagonals of $ABCD$ are not perpendicular	**A1**
		5
(b)	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. $\overrightarrow{AB}$ and $\overrightarrow{AD}$	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	**M1**
	Obtain answer 100.3°	**A1**
		3
(c)	Use a correct method to calculate the area, e.g. calculate $AB.AC$ sin BAD	**M1**
	Obtain answer 11.0 (**FT** on angle BAD)	**A1 FT**
		2

251. 9709_w20_MS_31 Q: 11

	Answer	Mark	Partial Marks
(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu$ $2 + 2\lambda = 1 - \mu$ $1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\frac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form $(12, -4, 4)$
		5	
(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) \mathbf{.} (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a - 3)^2 = 6(a^2 + 5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$ $- \lvert (a - 2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert . \lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a - 3)^2 = 6(a^2 + 5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	

252. 9709_w20_MS_32 Q: 8

<table>
<tr><th></th><th>Answer</th><th>Mark</th><th>Partial Marks</th></tr>
<tr><td rowspan="4">(a)</td><td>Obtain $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$</td><td>B1</td><td>Or equivalent seen or implied</td></tr>
<tr><td>Use the correct process for calculating the modulus of both vectors to obtain AB and CD</td><td>M1</td><td>$AB = \sqrt{24}$, $CD = \sqrt{6}$</td></tr>
<tr><td>Using exact values, verify that $AB = 2CD$</td><td>A1</td><td>Obtain given statement from correct work
Allow from $BA = 2DC$, OE</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="4">(b)</td><td>Use the correct process to calculate the scalar product of the relevant vectors (<i>their</i> $\overrightarrow{AB}$ and $\overrightarrow{CD}$)</td><td>M1</td><td>$\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$</td></tr>
<tr><td>Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result</td><td>M1</td><td></td></tr>
<tr><td>Obtain answer 99.6° (or 1.74 radians) or better</td><td>A1</td><td>Do not ISW if go on to subtract from 180°
(99.594…, 1.738...) Accept 260.4°</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="5">(c)</td><td>State correct vector equations for AB and CD in any form,
e.g. $(\mathbf{r} =) \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $(\mathbf{r} =) \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$</td><td>B1ft</td><td>Follow their $\overrightarrow{AB}$ and $\overrightarrow{CD}$
Alternative: $(\mathbf{r} =) \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -2 \\ -4 \end{pmatrix}$ and $(\mathbf{r} =) \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$</td></tr>
<tr><td>Equate at least two pairs of components of their lines and solve for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td>Obtain correct pair of values from correct equations</td><td>A1</td><td>Alternatives when taking A or B as point on line
<table>
<tr><td><i>A</i></td><td>λ</td><td>μ</td><td></td><td><i>B</i></td><td>λ</td><td>μ</td><td></td></tr>
<tr><td>ij</td><td>$-\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{17}{3} \neq \frac{7}{3}$</td><td>ij</td><td>$-\frac{7}{6}$</td><td>$-\frac{2}{3}$</td><td>$\frac{17}{3} \neq \frac{7}{3}$</td></tr>
<tr><td>ik</td><td>$\frac{1}{2}$</td><td>1</td><td>$0 \neq 2$</td><td>ik</td><td>$-\frac{1}{2}$</td><td>0</td><td>$0 \neq 2$</td></tr>
<tr><td>jk</td><td>$\frac{3}{2}$</td><td>-3</td><td>$5 \neq -5$</td><td>jk</td><td>$\frac{1}{2}$</td><td>-4</td><td>$5 \neq -5$</td></tr>
</table></td></tr>
<tr><td>Verify that all three equations are not satisfied and that the lines do not intersect</td><td>A1</td><td>CWO with conclusion
e.g. $\frac{17}{3} \neq \frac{7}{3}$ or $\frac{17}{3} = \frac{7}{3}$ is inconsistent or equivalent</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

253. 9709_m19_MS_32 Q: 9

	Answer	Mark	Partial Marks
(i)	State or imply a correct normal vector to either plane, e.g. $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, or $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
	Carry out correct process for evaluating the scalar product of two normal vectors	M1	
	Using the correct process for the moduli, divide the scalar product of the two normal vectors by the product of their moduli and evaluate the inverse cosine of the result	M1	
	Obtain answer 56.9° or 0.994 radians	A1	
		4	
(ii)	*EITHER*: Carry out a complete strategy for finding a point on the line (call the line l)	M1	
	Obtain such a point, e.g. (1, 1, 4)	A1	
	EITHER: State a correct equation for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l, e.g. $2a + 3b - c = 0$	B1	
	State a second equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1	
	Obtain $a : b : c = 1 : -3 : -7$, or equivalent	A1	
	State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$	A1	
	*OR*1: Attempt to calculate the vector product of the two normal vectors	M1	
	Obtain two correct components	A1	
	Obtain $\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$, or equivalent	A1	
	State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	A1	
(ii)	*OR*2: Obtain a second point on l e.g. (0, 4, 11)	B1	
	Subtract position vectors and obtain a direction vector for l	M1	
	Obtain $\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$, or equivalent	A1	
	State a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	A1	
	*OR*3: Express one variable in terms of a second	M1	
	Obtain a correct simplified expression, e.g. $y = 4 - 3x$	A1	
	Express the third variable in terms of the second	M1	
	Obtain a correct simplified expression, e.g. $z = 11 - 7x$	A1	
	Form a vector equation for the line	M1	
	State a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	A1	
		6	

	Answer		Mark	Partial Marks
(ii)	*OR*4:	Express one variable in terms of a second	**M1**	
		Obtain a correct simplified expression, e.g. $x = \frac{4}{3} - \frac{y}{3}$	**A1**	
		Express the same variable in terms of the third	**M1**	
		Obtain a correct simplified expression, e.g. $x = \frac{11}{7} - \frac{z}{7}$	**A1**	
		Form a vector equation for the line	**M1**	
		Obtain a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	**A1**	
			6	

254. 9709_s19_MS_31 Q: 9

	Answer	Mark	Partial Marks
(i)	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CB} = 2\mathbf{i} + \mathbf{j}$	**B1**	
	Use scalar product to obtain an equation in a, b, c,	**M1**	e.g. $2a + b = 0$, $a + 5c = 0$, $a + b - 5c = 0$
	Obtain two correct equations in a, b, c	**A1**	
	Solve to obtain $a : b : c$,	**M1**	or equivalent
	Obtain $a : b : c = 5 : -10 : -1$,	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent
	Alternative method 1		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CD} = \mathbf{i} + 5\mathbf{k}$	**B1**	$\overrightarrow{BD} = -\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
	Obtain a second such vector and calculate their vector product, e.g. $(2\mathbf{i} + \mathbf{j}) \times (\mathbf{i} + 5\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $5\mathbf{i} - 10\mathbf{j} - \mathbf{k}$	**A1**	
	Substitute to find d	**M1**	
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent
(i)	**Alternative method 2**		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{DB} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$	**B1**	
	Obtain a second such vector and form correctly a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} + 5\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	**A1**	
	State three equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
	Alternative method 3		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Substitute for D to obtain a third equation and eliminate one unknown (a, b, or d) entirely	**M1**	
	Obtain two correct equations in two unknowns, e.g. a, b, c	**A1**	
	Solve to obtain their ratio, e.g. $a : b : c$	**M1**	
	Obtain $a : b : c = 5 : -10 : -1$, $a : c : d = 5 : -1 : -25$, or $b : c : d = 10 : 1 : 25$	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent

	Answer	Mark	Partial Marks
(i)	**Alternative method 4**		
	Substitute for *B* and *C* and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Solve to obtain $a : b : d$	**M2**	or equivalent
	Obtain $a : b : d = 1 : -2 : -5$	**A1**	or equivalent
	Substitute for *C* to obtain *c*	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
		6	
(ii)	State or imply a normal vector for the plane *OABC* is **k**	**B1**	
	Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. $(5\mathbf{i} - 10\mathbf{j} - \mathbf{k}).(\mathbf{k})$	**M1**	i.e. correct process using **k** and their normal
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	Allow M1M1 for clear use of an incorrect vector that has been stated to be the normal to *OABC*
	Obtain answer 84.9° or 1.48 radians	**A1**	
		4	

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255. 9709_s19_MS_32 Q: 9

<table>
<tr><th></th><th>Answer</th><th>Mark</th><th>Partial Marks</th></tr>
<tr><td>(i)</td><td>Carry out correct method for finding a vector equation for <i>AB</i></td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain $(\mathbf{r} =)\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent</td><td>A1</td><td></td></tr>
<tr><td></td><td>Equate two pairs of components of general points on <i>their AB</i> and <i>l</i> and solve for λ or forμ</td><td>M1</td><td>$\begin{pmatrix} 1+2\lambda \\ 2-\lambda \\ -1+2\lambda \end{pmatrix} = \begin{pmatrix} 2+\mu \\ 1+\mu \\ 1+2\mu \end{pmatrix}$</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = 0$, $\mu = -1$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$)
Conclusion needs to follow correct values</td><td>A1</td><td>Alternatives
<table>
<tr><td><i>A</i></td><td>λ</td><td>μ</td><td></td><td><i>B</i></td><td>λ</td><td>μ</td><td></td></tr>
<tr><td>ij</td><td>$2/3$</td><td>$1/3$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td><td></td><td>$-1/3$</td><td>$1/3$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td></tr>
<tr><td>ik</td><td>0</td><td>–1</td><td>$2 \neq 0$</td><td></td><td>–1</td><td>–1</td><td>$2 \neq 0$</td></tr>
<tr><td>jk</td><td>1</td><td>0</td><td>$3 \neq 2$</td><td></td><td>0</td><td>0</td><td>$3 \neq 2$</td></tr>
</table></td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
<tr><td>(ii)</td><td>State or imply midpoint has position vector $2\mathbf{i} + \frac{3}{2}\mathbf{j}$</td><td>B1</td><td></td></tr>
<tr><td></td><td>Substitute in $2x - y + 2z = d$ and find <i>d</i></td><td>M1</td><td>Correct use of <i>their</i> direction for <i>AB</i> and <i>their</i> midpoint</td></tr>
<tr><td></td><td>Obtain plane equation $4x - 2y + 4z = 5$</td><td>A1</td><td>or equivalent e.g. $\mathbf{r}.\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \frac{5}{2}$</td></tr>
<tr><td></td><td>Substitute components of <i>l</i> in plane equation and solve for μ</td><td>M1</td><td>Correct use of their plane equation.</td></tr>
<tr><td></td><td>Obtain $\mu = -\frac{1}{2}$ and position vector $\frac{3}{2}\mathbf{i} + \frac{1}{2}\mathbf{j}$ for the point <i>P</i></td><td>A1</td><td>Final answer
Accept coordinates in place of position vector</td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
</table>

256. 9709_s19_MS_33 Q: 10

	Answer	Mark	Partial Marks
(i)	Find $\overrightarrow{PQ}$ for a general point Q on l, e.g. $-3\mathbf{i}+6\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$	B1	
	Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate the result to zero	M1	
	Solve for μ and obtain $\mu = 2$	A1	
	Carry out a complete method for finding the length of $\overrightarrow{PQ}$	M1	
	Obtain answer 3	A1	
	Alternative method for question 10(i)		
	Calling the point (1, 2, 3) A, state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form, e.g. 3**i** – 6**k**	B1	
	Use a scalar product with a direction vector for l to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) on l	M1	
	Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$	A1	
	Use Pythagoras to find the perpendicular	M1	
	Obtain answer 3	A1	
(i)	**Alternative method for question 10(i)**		
	State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form	B1	
	Calculate a vector product with a direction vector for l	M1	
	Obtain correct answer, e.g. 6**i** – 6**j** – 3**k**	A1	
	Divide modulus of the product by that of the direction vector	M1	
	Obtain answer 3	A1	
		5	
(ii)	Substitute coordinates of a general point of l in the plane equation and equate constant terms	M1	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	A1	
	Equate the coefficient of μ to zero	M1	
	Obtain a correct equation, e.g. $2a – b – 4 = 0$	A1	
	Substitute (1, 2, 3) in the plane equation	M1	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	A1	
	Alternative method for question 10(ii)		
	Find a second point on l and obtain an equation in a and/or b	M1	
	Obtain a correct equation, e.g. $5a – 2 = 13$	A1	
	Equate scalar product of a direction vector for l and a vector normal for the plane to zero	M1	
	Obtain a correct equation, e.g. $2a – b – 4 = 0$	A1	
	Solve for a or for b	M1	
	Obtain $a = 3$ and $b = 2$	A1	
		6	

257. 9709_w19_MS_31 Q: 7

	Answer	Mark	Partial Marks
(i)	Express general point of l or m in component form e.g. $(a + \lambda, 2 - 2\lambda, 3 + 3\lambda)$ or $(2 + 2\mu, 1 - \mu, 2 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain **either** $\lambda = -2$ or $\mu = -5$ **or** $\lambda = \frac{1}{3}a$ or $\mu = \frac{2}{3}a - 1$ **or** $\lambda = \frac{1}{5}(a-4)$ or $\mu = \frac{1}{5}(3a-7)$	**A1**	
	Obtain $a = -6$	**A1**	
		4	
(ii)	Use scalar product to obtain a relevant equation in a, b and c, e.g. $a - 2b + 3c = 0$	**B1**	
	Obtain a second equation, e.g. $2a - b + c = 0$ and solve for one ratio	**M1**	
	Obtain $a : b : c = 1 : 5 : 3$	**A1**	OE
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
	Alternative method for question 7(ii)		
	Attempt to calculate vector product of relevant vectors,	**M1**	e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}).(2\mathbf{i} - \mathbf{j} + \mathbf{k})$
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$	**A1**	
	Substitute a relevant point and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
(ii)	**Alternative method for question 7(ii)**		
	Using a relevant point and relevant vectors, form a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$	**A1FT**	
	State three correct equations in x, y, z, λ and μ	**A1FT**	
	Eliminate λ and μ	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
		5	

258. 9709_w19_MS_33 Q: 7

	Answer	Mark	Partial Marks
(i)	Substitute coordinates (5, 2, – 2) in $x + 4y - 8z = d$	**M1**	
	Obtain plane equation $x + 4y - 8z = 29$, or equivalent	**A1**	
		2	
(ii)	Attempt to use perpendicular formula to find perpendicular from (5, 2, – 2) to m	**M1**	
	Obtain a correct unsimplified expression, e.g. $\dfrac{5+8+16-2}{\sqrt{(1+16+64)}}$	**A1**	
	Obtain answer 3	**A1**	
	Alternative method 1 for question 7(ii)		
	State or imply perpendicular from O to m is $\frac{2}{9}$ or from O to n is $\frac{29}{9}$	**B1**	
	Find difference in perpendiculars	**M1**	
	Obtain answer 3	**A1**	
	Alternative method 2 for question 7(ii)		
	Obtain correct parameter value, or position vector or coordinates of the foot of the perpendicular from (5, 2, – 2) to m, e.g. $\mu = \pm\frac{1}{3}$; $\left(\frac{14}{3}, \frac{2}{3}, \frac{2}{3}\right)$	**B1**	
	Calculate the length of the perpendicular	**M1**	
	Obtain answer 3	**B1**	
		3	
(iii)	Calling the direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, use a scalar product to form a relevant equation in a, b and c, e.g. $a + 4b - 8c = 0$ or $5a + 2b - 2z = 0$	**B1**	
	Solve two relevant equations for the ratio $a : b : c$	**M1**	
	Obtain $a : b : c = 4 : -19 : -9$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
	Alternative method for question 7(iii)		
	Attempt to calculate vector product of two relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} - 8\mathbf{k})\times(5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain $8\mathbf{i} - 38\mathbf{j} - 18\mathbf{k}$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
		4	

259. 9709_m18_MS_32 Q: 10

	Answer	Mark
(i)	Express general point of l in component form, e.g. $\mathbf{r} = (4 + \mu)\mathbf{i} + (3 + 2\mu)\mathbf{j} + (-1 - 2\mu)\mathbf{k}$, or equivalent NB: Calling the vector $\mathbf{a} + \mu\mathbf{b}$, the **B1** is earned by a correct reduction of the sum to a single vector or by expressing the substitution as a distributed sum $\mathbf{a.n} + \mu\mathbf{b.n}$	**B1**
	Substitute in given equation of p and solve for μ	**M1**
	Obtain final answer 2**i** – **j** + 3**k** from $\mu = -2$	**A1**
		3

(ii)	Using the correct process, evaluate the scalar product of a direction vector for l and a normal for p	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	**M1**
	Obtain answer 10.3° (or 0.179 radians)	**A1**
		3
(iii)	*EITHER*: State $a + 2b - 2c = 0$ or $2a - 3b - c = 0$	**(B1**
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**
	Obtain $a : b : c = 8 : 3 : 7$, or equivalent	**A1**
	Substitute a, b, c and given point and evaluate d	**M1**
	Obtain answer $8x + 3y + 7z = 5$	**A1)**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})\times(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	**(M1**
	Obtain two correct components of the product	**A1**
	Obtain correct product, e.g. $8\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	**A1**
	Use the product and the given point to find d	**M1**
	Obtain answer $8x + 3y + 7z = 5$, or equivalent	**A1)**
	*OR*2: Attempt to form a 2-parameter equation with relevant vectors	**(M1**
	State a correct equation, e.g. $\mathbf{r} = 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) + \mu(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})$	**A1**
	State 3 equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	State answer $8x + 3y + 7z = 5$, or equivalent	**A1)**
		5

260. 9709_s18_MS_31 Q: 10

	Answer	Mark
(a)	*EITHER*: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l, e.g. $(1+\mu)\ \mathbf{i}+(4+2\mu)\mathbf{j}+(4+3\mu)\mathbf{k}$	**B1**
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero	**M1**
	Solve and obtain correct solution e.g. $\mu=-\frac{3}{2}$	**A1**
	Carry out method to calculate PQ	**M1**
	Obtain answer 1.22	**A1**
	*OR*1: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l	**B1**
	Use a correct method to express PQ^2 (or PQ) in terms of μ	**M1**
	Obtain a correct expression in any form	**A1**
	Carry out a complete method for finding its minimum	**M1**
	Obtain answer 1.22	**A1**
	*OR*2: Calling (4, 2, 5) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+4\mathbf{j}+4\mathbf{k}$	**B1**
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on l	**M1**
	Obtain correct answer $21/\sqrt{14}$, or equivalent	**A1**
	Use Pythagoras to find the perpendicular	**M1**
	Obtain answer 1.22	**A1**
	*OR*3: State $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form	**B1**
	Calculate vector product of $\overrightarrow{PA}$ and a direction vector for l	**M1**
	Obtain correct answer, e.g. $4\mathbf{i}+\mathbf{j}-2\mathbf{k}$	**A1**
	Divide modulus of the product by that of the direction vector	**M1**
	Obtain answer 1.22	**A1**
		5

	Answer	Mark
(ii)	*EITHER*: Use scalar product to obtain a relevant equation in a, b and c, e.g. $a + 2b + 3c = 0$	**B1**
	Obtain a second relevant equation, e.g. using $\overrightarrow{PA}$ $a + 4b + 4c = 0$, and solve for one ratio	**M1**
	Obtain $a : b : c = 4 : 1 : -2$, or equivalent	**A1**
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2k$	**A1**
	Substitute a relevant point and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*2: Using a relevant point and relevant vectors form a 2-parameter equation for the plane	**M1**
	State a correct equation, e.g. $\mathbf{r} = 4i + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	Obtain correct answer $4x + y - 2z = 8$, or equivalent	**A1**
		5

261. 9709_s18_MS_32 Q: 10

	Answer	Mark	Partial Marks
(i)	Equate at least two pairs of components and solve for *s* or for *t*	**M1**	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases}$ or $\begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases}$ or $\begin{cases} s = \frac{-2}{5} \\ t = \frac{-13}{5} \\ \frac{6}{5} \neq \frac{-8}{5} \end{cases}$
	Obtain correct answer for *s* or *t*, e.g. $s = -6$, $t = -11$	**A1**	
	Verify that all three equations are not satisfied and the lines fail to intersect	**A1**	
	State that the lines are not parallel	**B1**	
		4	
(ii)	*EITHER*: Use scalar product to obtain a relevant equation in *a*, *b* and *c*, e.g. $2a + 3b - c = 0$	**B1**	
	Obtain a second equation, e.g. $a + 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c$ and state correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, or equivalent	**A1**	
	OR: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})\times(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	**A1**	
		3	

	Answer	Mark	Partial Marks
(iii)	*EITHER*: State position vector or coordinates of the mid-point of a line segment joining points on *l* and *m*, e.g. $\frac{3}{2}\mathbf{i}+\mathbf{j}+\frac{5}{2}\mathbf{k}$	**B1**	*OR*: Use the result of **(ii)** to form equations of planes containing *l* and *m* B1
	Use the result of **(ii)** and the mid-point to find *d*	**M1**	Use average of distances to find equation of *p*. M1
	Obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	Obtain answer $5x - 3y + z = 7$, or equivalent A1
	OR: Using the result of part **(ii)**, form an equation in *d* by equating perpendicular distances to the plane of a point on *l* and a point on *m*	**M1**	
	State a correct equation, e.g. $\left\lvert\frac{14-d}{\sqrt{35}}\right\rvert = \left\lvert\frac{-d}{\sqrt{35}}\right\rvert$	**A1**	
	Solve for *d* and obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	
		3	

262. 9709_s18_MS_33 Q: 10

	Answer	Mark
(i)	Carry out a correct method for finding a vector equation for *AB*	**M1**
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{k})$, or equivalent	**A1**
	Equate pair(s) of components *AB* and *l* and solve for λ or μ	**M1(dep*)**
	Obtain correct answer for λ or μ	**A1**
	Verify that all three component equations are not satisfied	**A1**
	Total:	**5**
(ii)	State or imply a direction vector for *AP* has components $(2 + t, 5 + 2t, -3 - 2t)$	**B1**
	State or imply that cos 120° equals the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{AB}$ divided by the product of their moduli	**M1**
	Carry out the correct processes for finding the scalar product and the product of the moduli in terms of *t*, and obtain an equation in terms of *t*	**M1**
	Obtain the given equation correctly	**A1**
	Solve the quadratic and use a root to find a position vector for *P*	**M1**
	Obtain position vector $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{2}{3}$	**A1**
	Total:	**6**

263. 9709_w18_MS_31 Q: 10

	Answer	Mark	Partial Marks
(i)	*EITHER*: Expand scalar product of a normal to m and a direction vector of l	**M1**	
	Verify scalar product is zero	**A1**	
	Verify that one point of l does not lie in the plane	**A1**	
	OR: Substitute coordinates of a general point of l in the equation of the plane m	**M1**	
	Obtain correct equation in λ in any form	**A1**	
	Verify that the equation is not satisfied for any value of λ	**A1**	
		3	
(ii)	Use correct method to evaluate a scalar product of normal vectors to m and n	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 74.5° or 1.30 radians	**A1**	
		3	
(iii)	*EITHER*: Using the components of a general point P of l form an equation in λ by equating the perpendicular distance from n to 2	**M1**	
	OR: Take a point Q on l, e.g. (5, 3, 3) and form an equation in λ by equating the length of the projection of QP onto a normal to plane n to 2	**M1**	
	Obtain a correct modular or non-modular equation in any form	**A1**	
	Solve for λ and obtain a position vector for P, e.g. $7\mathbf{i} + 5\mathbf{j} + 7\mathbf{j}$ from $\lambda = 3$	**A1**	
	Obtain position vector of the second point, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ from $\lambda = -1$	**A1**	
		4	

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264. 9709_w18_MS_32 Q: 10

	Answer	Mark	Partial Marks
(i)	Substitute for $\mathbf{r}$ and expand the scalar product to obtain an equation in λ	**M1***	e.g. $3(5+\lambda)+(-3-2\lambda)+(-1+\lambda)=5 \ (2\lambda=5-11)$ or $3(4+\lambda)+1(-5-2\lambda)+(-1+\lambda)=0$ Must attempt to deal with $\mathbf{i} + 2\mathbf{j}$
	Solve a linear equation for λ	**M1(dep*)**	
	Obtain $\lambda = -3$ and position vector $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ for A	**A1**	Accept coordinates
		3	
(ii)	State or imply a normal vector of p is $3\mathbf{i} + \mathbf{j} + \mathbf{k}$, or equivalent	**B1**	
	Use correct method to evaluate a scalar product of relevant vectors e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}).(3\mathbf{i} + \mathbf{j} + \mathbf{k})$	**M1**	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\cos\theta = \frac{2}{\sqrt{6}\sqrt{11}}$ Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	**A1**	Or better

	Answer	Mark	Partial Marks
(ii)	**Alternative 1**		
	Use of a point on l and Cartesian equation $3x+y+z=5$ to find distance of point from plane e.g. $B(5,-3,-1)$ $d=\frac{3\times5-3-1-5}{\sqrt{9+1+1}}$	**M1**	
	$=\frac{6}{\sqrt{11}}\quad(=1.809...)$	**A1**	
	Complete method to find angle e.g. $\sin\theta=\frac{d}{AB}$	**M1**	
	$\theta=\sin^{-1}\left(\frac{6}{\sqrt{11}\sqrt{54}}\right)=0.249$	**A1**	Or better
	Alternative 2		
	State or imply a normal vector of p is $3\mathbf{i}+\mathbf{j}+\mathbf{k}$, or equivalent	**B1**	
	Use correct method to evaluate a vector product of relevant vectors e.g. $(\mathbf{i}-2\mathbf{j}+\mathbf{k})\mathbf{x}(3\mathbf{i}+\mathbf{j}+\mathbf{k})$	**M1**	$3\mathbf{i}-2\mathbf{j}+7\mathbf{k}$
	Using the correct process for calculating the moduli, divide the vector product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\sin\theta=\frac{\sqrt{3^2+2^2+7^2}}{\sqrt{11}\sqrt{6}}$. Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	**A1**	Or better
		4	

	Answer	Mark	Partial Marks
(iii)	Taking the direction vector of the line to be $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$, state a relevant equation in a, b, c, e.g. $3a+b+c=0$	**B1**	
	State a second relevant equation, e.g. $a-2b+c=0$, and solve for one ratio, e.g. $a:b$	**M1**	
	Obtain $a:b:c=3:-2:-7$, or equivalent	**A1**	
	State answer $\mathbf{r}=2\mathbf{i}+3\mathbf{j}-4\mathbf{k}+\mu\,(3\mathbf{i}-2\mathbf{j}-7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_A$ Requires '$\mathbf{r}=$ ….'
	Alternative		
	Attempt to calculate the vector product of relevant vectors, e.g. $(3\mathbf{i}+\mathbf{j}+\mathbf{k})\times(\mathbf{i}-2\mathbf{j}+\mathbf{k})$	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain correct product, e.g. $3\mathbf{i}-2\mathbf{j}-7\mathbf{k}$	**A1**	
	State answer $\mathbf{r}=2\mathbf{i}+3\mathbf{j}-4\mathbf{k}+\mu\,(3\mathbf{i}-2\mathbf{j}-7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_A$. Requires " $\mathbf{r}=$ …."
		4	

265. 9709_m17_MS_32 Q: 6

	Answer	Mark
(i)	Verify that the point with position vector $\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ lies in the plane	**B1**
	EITHER:	
	Find a second point on *l* and substitute its coordinates in the equation of *p*	**(M1**
	Verify that the second point, e.g. (3, 1, – 2), lies in the plane	**A1)**
	OR:	
	Expand scalar product of a normal to *p* and the direction vector of *l*	**(M1**
	Verify scalar product is zero	**A1)**
	Total:	**3**

	Answer	Mark
(ii)	*EITHER*:	
	Use scalar product to obtain a relevant equation in a, b and c, e.g. $2a-b+c=0$	**(B1**
	Obtain a second relevant equation, e.g. $3a+b-5c=0$, and solve for one ratio e.g. $a:b$	**M1**
	Obtain $a:b:c=4:13:5$, or equivalent	**A1**
	Substitute (3, – 1, 2) and the values of a, b and c in the general equation and find d	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*1:	
	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i}-\mathbf{j}+\mathbf{k})\times(3\mathbf{i}+\mathbf{j}-5\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i}+13\mathbf{j}+5\mathbf{k}$	**A1**
	Substitute (3, – 1, 2) in $4x+13y+5z=d$, or equivalent, and find d	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*2:	
	Using the relevant point and relevant vectors form a 2-parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+\mathbf{k})+\mu(3\mathbf{i}+\mathbf{j}-5\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*3:	
	Using the relevant point and relevant vectors form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & -1 & 1 \\ 3 & 1 & -5 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain or imply two correct cofactors	**A1**

	Answer	Mark
	Obtain answer $4x+13y+5z=9$, or equivalent	A1)
	Total:	**5**

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266. 9709_s17_MS_31 Q: 6

	Answer	Mark
(i)	State or obtain coordinates (1, 2, 1) for the mid-point of AB	**B1**
	Verify that the midpoint lies on m	**B1**
	State or imply a correct normal vector to the plane, e.g. $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$	**B1**
	State or imply a direction vector for the segment AB, e.g. $-4\mathbf{i}-4\mathbf{j}+2\mathbf{k}$	**B1**
	Confirm that m is perpendicular to AB	**B1**
	Total:	**5**
(ii)	State or imply that the perpendicular distance of m from the origin is $\frac{5}{3}$, or unsimplified equivalent	**B1**
	State or imply that n has an equation of the form $2x+2y-z=k$	**B1**
	Obtain answer $2x+2y-z=2$	**B1**
	Total:	**3**

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267. 9709_s17_MS_32 Q: 9

	Answer	Mark
(i)	*EITHER:* Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8+3\lambda, -3-\lambda, 4+2\lambda)$	**(B1**
	Equate scalar product of $\overrightarrow{AP}$ and direction vector of l to zero and solve for λ	**M1**
	Obtain $\lambda=-\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**A1)**

	Answer	Mark
	OR: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	(B1
	Differentiate $\|AP\|^2$ and solve for λ at minimum	M1
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	A1
	Carry out a complete method for finding the position vector of the reflection of A in l	M1
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	A1)
	Total:	**5**
(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $3a - b + 2c = 0$	(B1
	Form a second relevant equation, e.g. $9a - b + 8c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain final answer $a : b : c = 1 : 1 : -1$ and state plane equation $x + y - z = 0$	A1)
	*OR*1: Attempt to calculate vector product of two relevant vectors, e.g. $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$, and state plane equation $-x - y + z = 0$	A1)
	*OR*2: Using a relevant point and relevant vectors, attempt to form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 6\mathbf{i} + 6\mathbf{k} + s(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1
	State 3 correct equations in x, y, z, s and t	A1
	Eliminate s and t and state plane equation $x + y - z = 0$, or equivalent	A1)
	*OR*3: Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\begin{vmatrix} x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8 \end{vmatrix} = 0$	(M1
	Expand a correct determinant and obtain two correct cofactors	A1
	Obtain answer $-6x - 6y + 6z = 0$, or equivalent	A1)
	Total:	**3**

	Answer	Mark
(iii)	*EITHER*: Using the correct processes, divide the scalar product of $\overrightarrow{OA}$ and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula	(M1
	Obtain a correct expression in any form, e.g. $\frac{1+2-4}{\sqrt{(1^2+1^2+(-1)^2)}}$, or equivalent	A1 FT
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)
	OR1: Obtain equation of the parallel plane through *A*, e.g. $x+y-z=-1$ [The f.t. is on the plane found in part (**ii**).]	(B1 FT
	Use correct method to find its distance from the origin	M1
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)
	OR2: Form equation for the intersection of the perpendicular through *A* and the plane [FT on their **n**]	(B1 FT
	Solve for λ	M1
	$\lvert\lambda\mathbf{n}\rvert=\frac{1}{\sqrt{3}}$	A1)
	Total:	**3**

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268. 9709_s17_MS_33 Q: 10

	Answer	Mark
(i)	Carry out a correct method for finding a vector equation for *AB*	M1
	Obtain $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$, or equivalent	A1
	Equate two pairs of components of general points on *AB* and *l* and solve for λ or for μ	M1
	Obtain correct answer for λ or μ, e.g. $\lambda=\frac{5}{7}$ or $\mu=\frac{3}{7}$	A1
	Obtain $m=3$	A1
	Total:	**5**

	Answer	Mark
(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $a-2b-4c=0$	**(B1**
	Form a second relevant equation, e.g. $2a+3b-c=0$ and solve for one ratio, e.g. $a:b$	**M1**
	Obtain final answer $a:b:c=14:-7:7$	**A1**
	Use coordinates of a relevant point and values of a, b and c and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 1: Attempt to calculate the vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}-4\mathbf{k})\times(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $14\mathbf{i}-7\mathbf{j}+7\mathbf{k}$	**A1**
	Substitute coordinates of a relevant point in $14x-7y+7z=d$, or equivalent, and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 2: Using a relevant point and relevant vectors, form a 2–parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+s(\mathbf{i}-2\mathbf{j}-4\mathbf{k})+t(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**A1**
	State 3 correct equations in x, y, z, s and t	**A1**
	Eliminate s and t	**M1**
	Obtain answer $2x-y+z=6$, or equivalent	**A1)**
	OR 3: Using a relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & -2 & -4 \\ 2 & 3 & -1 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain or imply two correct cofactors	**A1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	Total:	**5**

269. 9709_w17_MS_31 Q: 10

	Answer	Mark
(i)	Equate at least two pairs of components of general points on l and m and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	**A1**
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	**A1**
		3
(ii)	Carry out correct process for evaluating scalar product of direction vectors for l and m	***M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**DM1**
	Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians	**A1**
		3
(iii)	*EITHER:* Use scalar product to obtain a relevant equation in a, b and c, e.g. $-a + b + 4c = 0$	**B1**
	Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$	**M1**
	Obtain $a : b : c = 2 : -2 : 1$, or equivalent	**A1**
	Substitute (3, −2, −1) and values of a, b and c in general equation and find d	**M1**
	Obtain answer $2x - 2y + z = 9$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	**A1**
	Substitute (3, −2, −1) in $-6x + 6y - 3z = d$, or equivalent, and find d	**M1**
	Obtain answer $-2x + 2y - z = -9$, or equivalent	**A1)**
	*OR*2: Using the relevant point and relevant vectors, form a 2-parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**

	Answer	Mark
	Obtain answer $2x - 2y + z = 9$, or equivalent	**A1)**
	*OR*3: Using the relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-3 & y+2 & z+1 \\ -1 & 1 & 4 \\ 2 & 1 & -2 \end{vmatrix} = 0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain two correct cofactors	**A1**
	Obtain answer $-2x + 2y - z = -9$, or equivalent	**A1)**
		5

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270. 9709_w17_MS_32 Q: 10

	Answer	Mark
(i)	State or imply a correct normal vector to either plane, e.g. $\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	**B1**
	Carry out correct process for evaluating the scalar product of two normal vectors	**M1**
	Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	**M1**
	Obtain final answer 72.5° or 1.26 radians	**A1**
		4
(ii)	*EITHER*: Substitute $y = 2$ in both plane equations and solve for x or for z	**(M1**
	Obtain $x = 3$ and $z = 1$	**A1)**
	OR: Find the equation of the line of intersection of the planes	
	Substitute $y = 2$ in line equation and solve for x or for z	**(M1**
	Obtain $x = 3$ and $z = 1$	**A1)**

	Answer	Mark
	EITHER: Use scalar product to obtain an equation in a, b and c, e.g. $a + b + 3c = 0$	**(B1**
	Form a second relevant equation, e.g. $2a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	***M1**
	Obtain final answer $a : b : c = 7 : 5 : -4$	**A1**
	Use coordinates of A and values of a, b and c in general equation and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*1: Calculate the vector product of relevant vectors, e.g. $(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**(*M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $7\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*2: Using relevant vectors, form a two-parameter equation for the plane	**(*M1**
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**A1 FT**
	State 3 correct equations in x, y, z, λ and μ	**A1 FT**
	Eliminate λ and μ	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*3: Use the direction vector of the line of intersection of the two planes as normal vector to the plane	**(*M1**
	Two correct components	**A1**
	Three correct components	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
		7

271. 9709_m16_MS_32 Q: 8

		Answer	Mark	
(i)	*EITHER*:	Substitute for **r** in the given equation of *p* and expand scalar product	**M1**	
		Obtain equation in λ in any correct form	**A1**	
		Verify this is not satisfied for any value of λ	**A1**	
	*OR*1:	Substitute coordinates of a general point of *l* in the Cartesian equation of plane *p*	**M1**	
		Obtain equation in λ in any correct form	**A1**	
		Verify this is not satisfied for any value of λ	**A1**	
	*OR*2:	Expand scalar product of the normal to *p* and the direction vector of *l*	**M1**	
		Verify scalar product is zero	**A1**	
		Verify that one point of *l* does not lie in the plane	**A1**	
	*OR*3:	Use correct method to find the perpendicular distance of a general point of *l* from *p*	**M1**	
		Obtain a correct unsimplified expression in terms of λ	**A1**	
		Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent, for all λ	**A1**	
	*OR*4:	Use correct method to find the perpendicular distance of a particular point of *l* from *p*	**M1**	
		Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent	**A1**	
		Show that the perpendicular distance of a second point is also $5/\sqrt{6}$, or equivalent	**A1**	[3]
(ii)	*EITHER*:	Calling the unknown direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ state equation $2a+b+3c=0$	**B1**	
		State equation $2a-b-c=0$	**B1**	
		Solve for one ratio, e.g. $a:b$	**M1**	
		Obtain ratio $a:b:c = 1:4:-2$, or equivalent	**A1**	
	OR:	Attempt to calculate the vector product of the direction vector of *l* and the normal vector of the plane *p*, e.g. $(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	**M2**	
		Obtain two correct components of the product	**A1**	
		Obtain answer $2\mathbf{i}+8\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
		Form line equation with relevant vectors	**M1**	
		Obtain answer $\mathbf{r}=5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mu(\mathbf{i}+4\mathbf{j}-2\mathbf{k})$, or equivalent	**A1**✓	[6]

272. 9709_s16_MS_31 Q: 9

	Answer	Mark
(i)	*EITHER*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$	B1
	Use scalar product to obtain an equation in a, b, c e.g. $a - 2b - 3c = 0$, $a + b - c = 0$, or $3b + 2c = 0$	M1
	State two correct equations	A1
	Solve to obtain ratio $a : b : c$	M1
	Obtain $a : b : c = 5 : -2 : 3$	A1
	Obtain equation $5x - 2y + 3z = 5$, or equivalent	A1
	*OR*1: Substitute for two points, e.g. A and B, and obtain $a + 3b + 2c = d$ and $2a + b - c = d$	(B1
	Substitute for another point, e.g. C, to obtain a third equation and eliminate one unknown entirely from all three equations	M1
	Obtain two correct equations in three unknowns, e.g. in a, b, c	A1
	Solve to obtain their ratio	M1
	Obtain $a : b : c = 5 : -2 : 3$, $a : c : d = 5 : 3 : 5$, $a : b : d = 5 : -2 : 5$, or $b : c : d = -2 : 3 : 5$	A1
	Obtain equation $5x - 2y + 3z = 5$, or equivalent	A1)
	*OR*2: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$	(B1
	Obtain a second such vector and calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \mathbf{k})$	M1
	Obtain two correct components of the product	A1
	Obtain correct answer e.g. $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	A1
	Substitute in $5x - 2y + 3z = d$ to find d	M1
	Obtain equation $5x - 2y + 3z = 5$, or equivalent	A1)
	*OR*3: Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{j} + 2\mathbf{k}$	(B1
	Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1
	Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k})$	A1
	State three correct equations in x, y, z, λ, μ	A1
	Eliminate λ and μ	M1
	Obtain equation $3x - 2y + 3z = 5$, or equivalent	A1)
		[6]
(ii)	Correctly form an equation for the line through D parallel to OA	M1
	Obtain a correct equation e.g. $\mathbf{r} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$	A1
	Substitute components in the equation of the plane and solve for λ	M1
	Obtain $\lambda = 2$ and position vector $-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$ for P	A1
	Obtain the given answer correctly	A1
		[5]

273. 9709_s16_MS_32 Q: 9

	Answer	Mark	
(i)	*Either* state or imply $\overrightarrow{AB}$ or $\overrightarrow{BC}$ in component form, *or* state position vector of midpoint of $\overrightarrow{AC}$	**B1**	
	Use a correct method for finding the position vector of *D*	**M1**	
	Obtain answer $3\mathbf{i}+3\mathbf{j}+\mathbf{k}$, or equivalent	**A1**	
	EITHER: Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. *AB* and *BC*	**M1**	
	Show that *ABCD* has a pair of adjacent sides that are equal	**A1**	
	OR: Calculate scalar product $\overrightarrow{AC}.\overrightarrow{BD}$ or equivalent	**M1**	
	Show that *ABCD* has perpendicular diagonals	**A1**	[5]
(ii)	*EITHER*: State $a+2b+3c=0$ or $2a+b-2c=0$	**B1**	
	Obtain two relevant equations and solve for one ratio, e.g. $a:b$	**M1**	
	Obtain $a:b:c=-7:8:-3$, or equivalent	**A1**	
	Substitute coordinates of a relevant point in $-7x+8y-3z=d$, and evaluate	**M1**	
	Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	
	*OR*1:Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}+2\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain correct product, e.g. $-7\mathbf{i}+8\mathbf{j}-3\mathbf{k}$	**A1**	
	Substitute coordinates of a relevant point in $-7x+8y-3z=d$ and evaluate d	**M1**	
	Obtain answer $-7x+8y-3z=29$ or equivalent	**A1**	
	*OR*2:Attempt to form a 2-parameter equation with relevant vectors	**M1**	
	State a correct equation, e.g. $\mathbf{r}=2\mathbf{i}+5\mathbf{j}-\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+\mu(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	**A1**	
	State 3 equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	
	*OR*3:Using a relevant point and relevant direction vectors, form a determinant equation for the plane	**M1**	
	State a correct equation, e.g. $\begin{vmatrix} x-2 & y-5 & z+1 \\ 1 & 2 & 3 \\ 2 & 1 & -2 \end{vmatrix}=0$	**A1**	
	Attempt to expand the determinant	**M1**	
	Obtain correct values of two cofactors	**A1**	
	Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	[5]

274. 9709_s16_MS_33 Q: 8

	Answer	Mark
(i)	State a correct equation for AB in any form, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent	**B1**
	Equate at least two pairs of components of AB and l and solve for λ or for μ	**M1**
	Obtain correct answer for λ or for μ, e.g. $\lambda = -1$ or $\mu = 2$	**A1**
	Show that not all three equations are not satisfied and that the lines do not intersect	**A1**
		[4]
(ii)	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l, e.g. $(1-\mu)\mathbf{i} + (-3+2\mu)\mathbf{j} + (-2+\mu)\mathbf{k}$	**B1**
	Calculate the scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	**M1**
	Solve and obtain $\mu = \frac{3}{2}$	**A1**
	Carry out a method to calculate AP when $\mu = \frac{3}{2}$	**M1**
	Obtain the given answer $\dfrac{1}{\sqrt{2}}$ correctly	**A1**
	OR 1:Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l	*(***B1**
	Use correct method to express AP^2 (or AP) in terms of μ	**M1**
	Obtain a correct expression in any form, e.g. $(1-\mu)^2 + (-3+2\mu)^2 + (-2+\mu)^2$	**A1**
	Carry out a complete method for finding its minimum	**M1**
	Obtain the given answer correctly	**A1)**
	OR 2:Calling (2, –2, –1) C, state $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form, e.g. $\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$	**(B1**
	Use a scalar product to find the projection of $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) on l	**M1**
	Obtain correct answer in any form, e.g. $\dfrac{9}{\sqrt{6}}$	**A1**
	Use Pythagoras to find the perpendicular	**M1**
	Obtain the given answer correctly	**A1)**
	OR 3:State $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form	**(B1**
	Calculate vector product of $\overrightarrow{AC}$ and a direction vector for l, e.g. $(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}) \times (-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**
	Obtain correct answer in any form, e.g. $\mathbf{i} + \mathbf{j} - \mathbf{k}$	**A1**
	Divide modulus of the product by that of the direction vector	**M1**
	Obtain the given answer correctly	**A1)**
		[5]

275. 9709_w16_MS_31 Q: 8

	Answer	Mark	
(i)	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$	**B1**	
	Use correct method to calculate their scalar product	**M1**	
	Show value is zero and planes are perpendicular	**A1**	[3]
(ii)	*EITHER*: Carry out a complete strategy for finding a point on l the line of intersection	**M1**	
	Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0)	**A1**	
	EITHER: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for l, e.g. $3a+b-c=0$ and $a-b+2c=0$	**B1**	
	Solve for one ratio, e.g. $a:b$	**M1**	
	Obtain $a:b:c=1:-7:-4$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
	*OR*1: Obtain a second point on l, e.g. (1, 0, 1)	**B1**	
	Subtract vectors and obtain a direction vector for l	**M1**	
	Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$	**A1**√	
	*OR*2: Attempt to find the vector product of the two normal vectors	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
	*OR*1: Express one variable in terms of a second variable	**M1**	
	Obtain a correct simplified expression, e.g. $y=7-7x$	**A1**	
	Express the third variable in terms of the second	**M1**	
	Obtain a correct simplified expression, e.g. $z=5-4x$	**A1**	
	Form a vector equation for the line	**M1**	
	Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
	*OR*2: Express one variable in terms of a second variable	**M1**	
	Obtain a correct simplified expression, e.g. $z=5-4x$	**A1**	
	Express the same variable in terms of the third	**M1**	
	Obtain a correct simplified expression e.g. $z=(7+4y)/7$	**A1**	
	Form a vector equation for the line	**M1**	
	Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	**A1**√	[6]

276. 9709_w16_MS_33 Q: 10

	Answer	Mark	
(i)	Express general point of *l* in component form e.g. $(1+2\lambda,\ 2-\lambda,\ 1+\lambda)$	**B1**	
	Using the correct process for the modulus form an equation in λ	**M1***	
	Reduce the equation to a quadratic, e.g. $6\lambda^2+2\lambda-4=0$	**A1**	
	Solve for λ (usual requirements for solution of a quadratic)	**DM1**	
	Obtain final answers $-\mathbf{i}+3\mathbf{j}$ and $\frac{7}{3}\mathbf{i}+\frac{4}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$	**A1**	[5]
(ii)	Using the correct process, find the scalar product of a direction vector for *l* and a normal for *p*	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$	**M1**	
	State a correct equation in any form, e.g. $\dfrac{2a-1+1}{\sqrt{(a^2+1+1)}.\sqrt{(2^2+(-1)^2+1)}}=\pm\dfrac{2}{3}$	**A1**	
	Solve for a^2	**M1**	
	Obtain answer $a=\pm 2$	**A1**	[5]

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277. 9709_s15_MS_31 Q: 6

	Answer	Mark	
(i)	Obtain $\pm\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ as direction vector of l_1	B1	
	State that two direction vectors are not parallel	B1	
	Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1-3\lambda, 5-4\lambda)$ or $(7+\mu, l+2\mu, 1+5\mu)$	B1	
	Equate at least two pairs of components and solve for λ or for μ	M1	
	Obtain correct answers for λ and μ	A1	
	Verify that all three component equations are not satisfied (with no errors seen)	A1	[6]
(ii)	Carry out correct process for evaluating scalar product of $\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $\begin{pmatrix}1\\0\\0\end{pmatrix}$	M1	
	Use correct process for finding modulus and evaluating inverse cosine	M1	
	Obtain 79.5° or 1.39 radians	A1	[3]

278. 9709_s15_MS_32 Q: 10

		Answer	Mark	
(i)		Carry out a correct method for finding a vector equation for *AB*	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	A1	
		Equate at least two pairs of components of general points on *AB* and *l* and solve for λ or for μ	M1	
		Obtain correct answer for λ or μ, e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$; or $\lambda = \frac{1}{4}$ or $\mu = -\frac{3}{2}$	A1	
		Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1	**[5]**
(ii)	*EITHER*:	Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Use scalar product to obtain an equation in *a*, *b* and *c*, e.g. $3a + b - c = 0$	B1	
		Form a second relevant equation, e.g. $a - 2b + c = 0$ and solve for one ratio, e.g. $a : b$	M1	
		Obtain final answer $a : b : c = 1 : 4 : 7$ A1		
		Use coordinates of a relevant point and values of *a*, *b* and *c* in general equation and find *d*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
	*OR*1:	Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} - \mathbf{k})$	M1	
		Obtain two correct components	A1	
		Obtain correct answer, e.g. $\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $x + 4y + 7z = d$, or equivalent, and find *d*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
	*OR*2:	Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a 2-parameter equation for the plane	M1	
		State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(3\mathbf{i} + \mathbf{j} - \mathbf{k})$	A1	
		State 3 correct equations in *x*, *y*, *z*, *s* and *t*	A1	
		Eliminate *s* and *t*	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
	*OR*3:	Using the coordinates of *A* and two points on *l*, state three simultaneous equations in *a*, *b*, *c* and *d*, e.g. $a + b + 2c = d$, $2a - b + 3c = d$ and $4a + 2b + c = d$	B1	
		Solve and find one ratio, e.g. $a : b$	M1	
		State one correct ratio	A1	
		Obtain a correct ratio of three of the unknowns, e.g. $a : b : c = 1 : 4 : 7$, or equivalent	A1	
		Either use coordinates of a relevant point and the found ratio to find the fourth unknown, e.g. *d*, or find the ratio $a : b : c : d$	M1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	
	*OR*4:	Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	B1	
		Using a relevant point and second relevant vector, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-2 & y+1 & z-3 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 0$	A1	
		Attempt to expand the determinant	M1	
		Obtain or imply two correct cofactors	A1	
		Obtain answer $x + 4y + 7z = 19$, or equivalent	A1	**[6]**

279. 9709_s15_MS_33 Q: 9

	Answer	Mark	
(i)	State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$	B1	
	Carry out correct process for evaluating the scalar product of two normal vectors	M1	
	Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1	
	Obtain answer 85.9° or 1.50 radians	A1	**4**

		Answer	Mark	
(ii)	*EITHER*:	Carry out a complete strategy for finding a point on *l*	M1	
		Obtain such a point, e.g. (0, 2, 1)	A1	
		EITHER: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for *l*,		
		e.g. $a + 3b - 2c = 0$		
		and $2a + b + 3c = 0$	B1	
		Solve for one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c = 11 : -7 : -5$	A1	
		State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda(11\mathbf{i}-7\mathbf{j}-5\mathbf{k})$	A1√	
		*OR*1: Obtain a second point on *l*, e.g. $\left(\frac{22}{7}, 0, -\frac{3}{7}\right)$	B1	
		Subtract position vectors and obtain a direction vector for *l*	M1	
		Obtain $22\mathbf{i}-14\mathbf{j}-10\mathbf{k}$, or equivalent	A1	
		State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda(22\mathbf{i}-14\mathbf{j}-10\mathbf{k})$	A1√	
		*OR*2: Attempt to find the vector product of the two normal vectors	M1	
		Obtain two correct components	A1	
		Obtain $11\mathbf{i}-7\mathbf{j}-5\mathbf{k}$, or equivalent	A1	
		State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda(11\mathbf{i}-7\mathbf{j}-5\mathbf{k})$	A1√	
	*OR*3:	Express one variable in terms of a second	M1	
		Obtain a correct simplified expression, e.g. $x=(22-11y)/7$	A1	
		Express the same variable in terms of the third	M1	
		Obtain a correct simplified expression, e.g. $x=(11-11z)/5$	A1	
		Form a vector equation for the line M1		
		State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda\left(\mathbf{i}-\frac{7}{11}\mathbf{j}-\frac{5}{11}\mathbf{k}\right)$	A1√	
	*OR*4:	Express one variable in terms of a second	M1	
		Obtain a correct simplified expression, e.g. $y=(22-7x)/11$	A1	
		Express the third variable in terms of the second	M1	
		Obtain a correct simplified expression, e.g. $z=(11-5x)/11$	A1	
		Form a vector equation for the line	M1	
		State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda\left(\mathbf{i}-\frac{7}{11}\mathbf{j}-\frac{5}{11}\mathbf{k}\right)$	A1√	**6**
		[The √ marks are dependent on all M marks being earned.]		

280. 9709_w15_MS_31 Q: 7

	Answer	Mark	
(i)	Use correct method to form a vector equation for *AB*	M1	
	Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1	[2]
(ii)	Using a direction vector for *AB* and a relevant point, obtain an equation for *m* in any form	M1	
	Obtain answer $2x - 2y + z = 4$, or equivalent	A1	[2]
(iii)	Express general point of *AB* in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$	B1√	
	Substitute in equation of *m* and solve for λ or for μ	M1	
	Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of *N*, from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$	A1	
	Carry out a correct method for finding *CN*	M1	
	Obtain the given answer $\sqrt{13}$	A1	[5]
	[The f.t. is on the direction vector for *AB*.]		

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281. 9709_w15_MS_33 Q: 8

	Answer	Mark	
(i)	Express a general point on the line in single component form, e.g. $(\lambda, 2 - 3\lambda, -8 + 4\lambda)$, substitute in equation of plane and solve for λ	M1	
	Obtain $\lambda = 3$	A1	
	Obtain $(3, -7, 4)$	A1	[3]
(ii)	State or imply normal vector to plane is $4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$	B1	
	Carry out process for evaluating scalar product of two relevant vectors	M1	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate $\sin^{-1}$ or $\cos^{-1}$ of the result.	M1	
	Obtain $54.8°$ or 0.956 radians	A1	[4]
(iii)	<u>Either</u> Find at least one position of *C* by translating by appropriate multiple of direction vector $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ from *A* or *B*	M1	
	Obtain $(-3, 11, -20)$	A1	
	Obtain $(9, -25, 28)$	A1	
	<u>Or</u> Form quadratic equation in λ by considering $BC^2 = 4AB^2$	M1	
	Obtain $26\lambda^2 - 156\lambda - 702 = 0$ or equivalent and hence $\lambda = -3, \lambda = 9$	A1	
	Obtain $(-3, 11, -20)$ and $(9, -25, 28)$	A1	[3]

282. 9709_s20_MS_31 Q: 8

(a)	State $\frac{\mathrm{d}y}{\mathrm{d}x}=k\frac{y}{x\sqrt{x}}$, or equivalent	B1
	Separate variables correctly and attempt integration of at least one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $-2k\frac{1}{\sqrt{x}}$, or equivalent	A1
	Use given coordinates to find k or a constant of integration c in a solution containing terms of the form $a\ln y$ and $\frac{b}{\sqrt{x}}$, where $ab\neq 0$	M1
	Obtain $k=1$ and $c=2$	A1 + A1
	Obtain final answer $y=\exp\left(-\frac{2}{\sqrt{x}}+2\right)$, or equivalent	A1
		8
(b)	State that y approaches e^2 (FT *their* c in part **(a)** of the correct form)	B1FT
		1

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283. 9709_s20_MS_32 Q: 7

	Separate variables correctly and integrate at least one side	B1
	Obtain term $\ln(y-1)$	B1
	Carry out a relevant method to determine A and B such that $\frac{1}{(x+1)(x+3)}\equiv\frac{A}{x+1}+\frac{B}{x+3}$	M1
	Obtain $A=\frac{1}{2}$ and $B=-\frac{1}{2}$	A1
	Integrate and obtain terms $\frac{1}{2}\ln(x+1)-\frac{1}{2}\ln(x+3)$ $\frac{1}{2}\ln(x+1)-\frac{1}{2}\ln(x+3)$, or equivalent (**FT** is on A and B)	A1 FT + A1 FT
	Use $x=0$, $y=2$ to evaluate a constant, or as limits in a solution containing terms of the form $a\ln(y-1)$, $b\ln(x+1)$ and $c\ln(x+3)$, where $abc\neq 0$	M1
	Obtain correct answer in any form	A1
	Obtain final answer $y=1+\sqrt{\left(\frac{3x+3}{x+3}\right)}$, or equivalent	A1
		9

284. 9709_s20_MS_33 Q: 10

(a)	State or imply $\frac{dV}{dt} = -k\sqrt{h}$	**B1**
	State or imply $\frac{dV}{dh} = 2\pi rh - \pi h^2$, or equivalent	**B1**
	Use $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$	**M1**
	Obtain the **given answer** correctly	**A1**
		4
(b)	Separate variables and attempt integration of at least one side	**M1**
	Obtain terms $\frac{4}{3}rh^{\frac{3}{2}} - \frac{2}{5}h^{\frac{5}{2}}$ and $-Bt$	**A3, 2, 1, 0**
	Use $t = 0$, $h = r$ to find a constant of integration c	**M1**
	Use $t = 14$, $h = 0$ to find B	**M1**
	Obtain correct c and B, e.g. $c = \frac{14}{15}r^{\frac{5}{2}}$, $B = \frac{1}{15}r^{\frac{5}{2}}$	**A1**
	Obtain final answer $t = 14 - 20\left(\frac{h}{r}\right)^{\frac{3}{2}} + 6\left(\frac{h}{r}\right)^{\frac{5}{2}}$, or equivalent	**A1**
		8

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285. 9709_w20_MS_31 Q: 8

	Answer	**Mark**	**Partial Marks**
	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}dy = \frac{1-2x^2}{x}dx$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

286. 9709_w20_MS_32 Q: 7

	Answer	Mark	Partial Marks
(a)	Correct separation of variables	**B1**	$\int \sec^2 2x \, dx = \int e^{-3t} dt$ Needs correct structure
	Obtain term $-\frac{1}{3}e^{-3t}$	**B1**	
	Obtain term of the form $k \tan 2x$	**M1**	From correct working
	Obtain term $\frac{1}{2}\tan 2x$	**A1**	
	Use $x = 0$, $t = 0$ to evaluate a constant, or as limits in a solution containing terms of the form $a \tan 2x$ and be^{-3t}, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	e.g. $\frac{1}{2}\tan 2x = -\frac{1}{3}e^{-3t} + \frac{1}{3}$
	Obtain final answer $x = \frac{1}{2}\tan^{-1}\left(\frac{2}{3}\left(1 - e^{-3t}\right)\right)$	**A1**	
		7	
(b)	State that x approaches $\frac{1}{2}\tan^{-1}\left(\frac{2}{3}\right)$	**B1 FT**	Correct value. Accept $x \to 0.294$ The FT is dependent on letting $e^{-3t} \to 0$ in a solution containing e^{-3t}.
		1	

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287. 9709_m19_MS_32 Q: 6

	Answer	Mark	Partial Marks
	Separate variables correctly and attempt integration of at least one side	**B1**	
	Obtain term $-\frac{1}{2y^2}$, or equivalent	**B1**	
	Obtain term $-k\ e^{-x}$	**B1**	
	Use a pair of limits, e.g. $x = 0$, $y = 1$ to obtain an equation in k and an arbitrary constant c	**M1**	
	Use a second pair of limits, e.g. $x = 1$, $y = \sqrt{e}$, to obtain a second equation and solve for k or for c	**M1**	
	Obtain $k = \frac{1}{2}$ and $c = 0$	**A1**	
	Obtain final answer $y = e^{\frac{1}{2}x}$, or equivalent	**A1**	
		7	

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288. 9709_s19_MS_31 Q: 5

	Answer	Mark	Partial Marks
(i)	Use chain rule	**M1**	$k\cos\theta\sin^{-3}\theta\left(=-k\operatorname{cosec}^2\theta\cot\theta\right)$ Allow M1 for $-2\cos\theta\sin^{-1}\theta$
	Obtain correct answer in any form	**A1**	e.g. $-2\operatorname{cosec}^2\theta\cot\theta$, $\frac{-2\cos\theta}{\sin^3\theta}$ Accept $\frac{-2\sin\theta\cos\theta}{\sin^4\theta}$
		2	
(ii)	Separate variables correctly and integrate at least one side	**B1**	$\int x\,\mathrm{d}x=\int-\operatorname{cosec}^2\theta\cot\theta\,\mathrm{d}\theta$
	Obtain term $\frac{1}{2}x^2$	**B1**	
	Obtain term of the form $\frac{k}{\sin^2\theta}$	**M1***	or equivalent
	Obtain term $\frac{1}{2\sin^2\theta}$	**A1**	or equivalent
	Use $x = 4$, $\theta=\frac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution with terms ax^2 and $\frac{b}{\sin^2\theta}$, where $ab \neq 0$	**DM1**	Dependent on the preceding M1
	Obtain solution $x=\sqrt{\left(\operatorname{cosec}^2\theta+12\right)}$	**A1**	or equivalent
		6	

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289. 9709_s19_MS_32 Q: 7

	Answer	Mark	Partial Marks
(i)	Separate variables correctly and attempt integration of at least one side	**B1**	$\int e^{-y}\,\mathrm{d}y=\int xe^x\,\mathrm{d}x$
	Obtain term $-e^{-y}$	**B1**	B0B1 is possible
	Commence integration by parts and reach $xe^x\pm\int e^x\,\mathrm{d}x$	**M1**	B0B0M1A1 is possible
	Obtain xe^x-e^x	**A1**	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0$, $y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y}, bxe^x and ce^x, where $abc \neq$ 0	**M1**	Must see this step
	Obtain correct solution in any form	**A1**	e.g. $e^{-y}=e^x-xe^x$
	Rearrange as $y=-\ln(1-x)-x$	**A1**	or equivalent e.g. $y=\ln\frac{1}{e^x(1-x)}$ ISW
		7	
(ii)	Justify the given statement	**B1**	e.g. require $1-x>0$ for the ln term to exist, hence $x<1$ Must be considering the range of values of x, and must be relevant to *their* y involving $\ln(1-x)$
		1	

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290. 9709_s19_MS_33 Q: 5

	Answer	Mark	Partial Marks
	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term $\ln(x+1)$	**B1**	
	Obtain term of the form $a\ln(y^2+5)$	**M1**	
	Obtain term $\frac{1}{2}\ln(y^2+5)$	**A1**	
	Use $y=2$, $x=0$ to determine a constant, or as limits, in a solution containing terms $a\ln(y^2+5)$ and $b\ln(x+1)$, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	
	Obtain final answer $y^2=9(x+1)^2-5$	**A1**	
		7	

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291. 9709_w19_MS_31 Q: 4

	Answer	Mark	Partial Marks
(i)	State $\frac{dN}{dt} = ke^{-0.02t}N$ and show $k = -0.01$	**B1**	OE $(-10 = k\times1\times1000)$
		1	
(ii)	Separate variables correctly and integrate at least one side	**B1**	$\int\frac{1}{N}dN = \int -0.01e^{-0.02t}dt$
	Obtain term $\ln N$	**B1**	OE
	Obtain term $0.5e^{-0.02t}$	**B1**	OE
	Use $N=1000$, $t=0$ to evaluate a constant, or as limits, in a solution with terms $a\ln N$ and $be^{-0.02t}$, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5\left(e^{-0.02t}-1\right)$	**A1**	$\ln 1000 - \frac{1}{2} = 6.41$
	Substitute $N=800$ and obtain $t=29.6$	**A1**	
		6	
(iii)	State that N approaches $\frac{1000}{\sqrt{e}}$	**B1**	Accept 606 or 607 or 606.5
		1	

292. 9709_w19_MS_32 Q: 6

	Answer	Mark	Partial Marks
	Separate variables correctly to obtain $\int \frac{1}{x+2}\,dx = \int \cot\frac{1}{2}\theta \, d\theta$	**B1**	Or equivalent integrands. Integral signs SOI
	Obtain term $\ln(x+2)$	**B1**	Modulus signs not needed.
	Obtain term of the form $k\ln\sin\frac{1}{2}\theta$	**M1**	
	Obtain term $2\ln\sin\frac{1}{2}\theta$	**A1**	
	Use $x = 1$, $\theta = \frac{1}{3}\pi$ to evaluate a constant, or as limits, in an expression containing $p\ln(x+2)$ and $q\ln\left(\sin\frac{1}{2}\theta\right)$	**M1**	Reach $C =$ an expression or a decimal value
	Obtain correct solution in any form e.g. $\ln(x+2) = 2\ln\sin\frac{1}{2}\theta + \ln 12$	**A1**	$\ln 12 = 2.4849$….. Accept constant to at least 3 s.f. Accept with $\ln 3 - 2\ln\frac{1}{2}$
	Remove logarithms and use correct double angle formula	**M1**	Need correct algebraic process. $\left(\frac{x+2}{12} = \frac{1-\cos\theta}{2}\right)$
	Obtain answer $x = 4 - 6\cos\theta$	**A1**	
		8	

293. 9709_w19_MS_32 Q: 10

	Answer	Mark	Partial Marks
(i)	Express general point of l in component form e.g. $(1+\lambda, 3-2\lambda, -2+3\lambda)$	B1	
	Substitute in equation of p and solve for λ	M1	
	Obtain final answer $\frac{5}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}$ from $\lambda=\frac{2}{3}$	A1	OE Accept $1.67\mathbf{i}+1.67\mathbf{j}$ or better
		3	
(ii)	Use correct method to evaluate a scalar product of relevant vectors e.g. $(\mathbf{i}-2\mathbf{j}+3\mathbf{k}).(2\mathbf{i}+\mathbf{j}-3\mathbf{k})$	M1	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	M1	$\lvert\sin\theta\rvert=\frac{9}{14}$
	Obtain answer 40.0° or 0.698 radians	A1	AWRT
		3	
	Alternative method for question 10(ii)		
	Use correct method to evaluate a vector product of relevant vectors e.g. $(\mathbf{i}-2\mathbf{j}+3\mathbf{k})\text{x}(2\mathbf{i}+\mathbf{j}-3\mathbf{k})$	M1	
	Using the correct process for calculating the moduli, divide the modulus of the vector product by the product of the moduli of the two vectors and evaluate the inverse sine or cosine of the result	M1	$\cos\theta=\frac{\sqrt{115}}{14}$
	Obtain answer 40.0° or 0.698 radians	A1	AWRT
		3	
(iii)	State $a-2b+3c=0$ or $2a+b-3c=0$	B1	
	Obtain two relevant equations and solve for one ratio, e.g. $a:b$	M1	Could use $2a+b-3c=0$ and $\begin{cases} a+3b-2c=d \\ \frac{5}{3}a+\frac{5}{3}b=d \end{cases}$ i.e. use two points on the line rather than the direction of the line. The second M1 is not scored until they solve for d.
	Obtain $a:b:c=3:9:5$	A1	OE
	Substitute a, b, c and a relevant point in the plane equation and evaluate d	M1	Using their calculated normal and a relevant point
	Obtain answer $3x+9y+5z=20$	A1	OE
	Alternative method for question 10(iii)		
	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}+\mathbf{j}-3\mathbf{k})$	M1	
	Obtain two correct components	A1	
	Obtain correct answer, e.g. $3\mathbf{i}+9\mathbf{j}+5\mathbf{k}$	A1	
	Use the product and a relevant point to find d	M1	Using *their* calculated normal and a relevant point
	Obtain answer $3x+9y+5z=20$, or equivalent	A1	OE
(iii)	**Alternative method for question 10(iii)**		
	Attempt to form a 2-parameter equation with relevant vectors	M1	
	State a correct equation e.g. $\mathbf{r}=\mathbf{i}+3\mathbf{j}-2\mathbf{k}+\lambda(\mathbf{i}-2\mathbf{j}+3\mathbf{k})+\mu(2\mathbf{i}+\mathbf{j}-3\mathbf{k})$	A1	
	State 3 equations in x, y, z, λ and μ	A1	
	Eliminate λ and μ	M1	
	Obtain answer $3x+9y+5z=2$	A1	OE
		5	

294. 9709_w19_MS_33 Q: 9

	Answer	Mark	Partial Marks
(i)	Separate variables correctly and integrate one side	B1	
	Obtain term 0.2*t*, or equivalent	B1	
	Carry out a relevant method to obtain *A* and *B* such that $\frac{1}{(20-x)(40-x)} \equiv \frac{A}{20-x} + \frac{B}{40-x}$	*M1	OE
	Obtain $A = \frac{1}{20}$ and $B = -\frac{1}{20}$	A1	
	Integrate and obtain terms $-\frac{1}{20}\ln(20-x)+\frac{1}{20}\ln(40-x)$ OE	A1FT +A1FT	The FT is on *A* and *B*
	Use $x = 10$, $t = 0$ to evaluate a constant, or as limits	DM1	
	Obtain correct answer in any form	A1	
	Obtain final answer $x = \frac{60e^{4t}-40}{3e^{4t}-1}$	A1	OE
		9	
(ii)	State that *x* approaches 20	B1	
		1	

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295. 9709_m18_MS_32 Q: 6

	Answer	Mark
(i)	Show sufficient working to justify the given statement **AG**	B1
		1
(ii)	Separate variables correctly and attempt integration of at least one side	B1
	Obtain term $\frac{1}{2}x^2$	B1
	Obtain terms $\tan^2\theta + \tan\theta$, or $\sec^2\theta + \tan\theta$	B1 + B1
	Evaluate a constant, or use limits $x = 1,\ \theta = \frac{1}{4}\pi$, in a solution with two terms of the form ax^2 and $b\tan\theta$, where $ab \neq 0$	M1
	State correct answer in any form, e.g. $\frac{1}{2}x^2 = \tan^2\theta + \tan\theta - \frac{3}{2}$	A1
	Substitute $\theta = \frac{1}{3}\pi$ and obtain $x = 2.54$	A1
		7

296. 9709_s18_MS_31 Q: 6

	Answer	Mark
(i)	Separate variables correctly and integrate at least one side	**B1**
	Obtain term ln x	**B1**
	Obtain term $-\frac{2}{3}kt\sqrt{t}$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 100$ and $t = 0$, in a solution containing terms a ln x and $bt\sqrt{t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln x = -\frac{2}{3}kt\sqrt{t} + \ln 100$	**A1**
		5
(ii)	Substitute $x = 80$ and $t = 25$ to form equation in k	**M1**
	Substitute $x = 40$ and eliminate k	**M1**
	Obtain answer $t = 64.1$	**A1**
		3

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297. 9709_s18_MS_32 Q: 3

	Answer	Mark	Partial Marks
(i)	Fully justify the given statement	**B1**	Some indication of use of gradient of curve = gradient of tangent (*PT*) and no errors seen /no incorrect statements
		1	
(ii)	Separate variables and attempt integration of at least one side Obtain terms ln y and $\frac{1}{2}x$	**B1 B1**	Must be working from $\int \frac{1}{y}\,dy = \int k\,dx$ B marks are not available for fortuitously correct answers
	Use $x = 4$, $y = 3$ to evaluate a constant or as limits in a solution with terms a ln y and bx, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	$\ln y = \frac{1}{2}x + \ln 3 - 2$
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	**A1**	Accept $y = e^{\frac{1}{2}x+\ln 3-2}$, $y = e^{\frac{x-1.80}{2}}$, $y = 3\sqrt{e^{x-4}}$ $\lvert y\rvert = \ldots$ scores A0
		5	

298. 9709_s18_MS_33 Q: 6

	Answer	Mark
(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	M1
	Obtain $A = B = \frac{1}{4}$	A1
	Total:	2
(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln(2+y)$ or $c \ln(2-y)$	M1
	Obtain term $\ln x$	B1
	Integrate and obtain terms $\frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y)$	A1FT
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln(2+y)$ and $c \ln(2-y)$	M1
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y) - \frac{1}{4}\ln 3$	A1
	Rearrange as $\frac{2(3x^4-1)}{(3x^4+1)}$, or equivalent	A1
	Total:	6

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299. 9709_w18_MS_31 Q: 5

	Answer	Mark	Partial Marks
	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term $\ln y$	**B1**	
	Obtain terms $2 \ln x - \frac{1}{2}x^2$	**B1+B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \frac{1}{2}x^2 + \frac{1}{2}$	**A1**	
	Rearrange as $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$, or equivalent	**A1**	
		7	

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300. 9709_w18_MS_32 Q: 6

	Answer	Mark	Partial Marks
	State equation $\frac{\mathrm{d}y}{\mathrm{d}x} = k\frac{y^2}{x}$, or equivalent	**B1**	SC: If $k = 1$ seen or implied give B0 and then allow B1B1B0M1, max 3/8.
	Separate variables correctly and integrate at least one side	**B1**	$\int \frac{k}{x}\mathrm{d}x = \int \frac{1}{y^2}\mathrm{d}y$ Allow with incorrect value substituted for k
	Obtain terms $-\frac{1}{y}$ and $k \ln x$	**B1 + B1**	Incorrect k used scores max. B1B0
	Use given coordinates correctly to find k and/or a constant of integration C in an equation containing terms $\frac{a}{y}$, $b\ln x$ and C	**M1**	SC: If an incorrect method is used to find k, M1 is allowable for a correct method to find C
	Obtain $k = \frac{1}{2}$ and $c = -1$, or equivalent	**A1 + A1**	$\frac{1}{2}\ln x = 1 - \frac{1}{y}$ A0 for fortuitous answers.
	Obtain answer $y = \frac{2}{2 - \ln x}$, or equivalent, and ISW	**A1**	$y = \frac{-1}{-1 + \ln\sqrt{x}}$
			SC: MR of the fraction. $\frac{\mathrm{d}y}{\mathrm{d}x} = k\frac{y^2}{x^2}$ B1 Separate variables and integrate B1 $\frac{-1}{y} = \frac{-k}{x}(+C)$ B1+B1 Substitute to find k and/or c M1 $k = \frac{\mathrm{e}}{2(\mathrm{e}-1)}$, $c = \frac{2-\mathrm{e}}{2(\mathrm{e}-1)}$ A1+A1 Answer A0
		8	

301. 9709_s17_MS_31 Q: 9

	Answer	Mark
(i)	Carry out a relevant method to obtain A and B such that $\frac{1}{x(2x+3)} \equiv \frac{A}{x} + \frac{B}{2x+3}$, or equivalent	M1
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	A1
	Total:	2
(ii)	Separate variables and integrate one side	B1
	Obtain term $\ln y$	B1
	Integrate and obtain terms $\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3)$, or equivalent	B2 FT
	Use x = 1 and y = 1 to evaluate a constant, or as limits, in a solution containing $a\ln y,\ b\ln x,\ c\ln(2x+3)$	M1
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) + \frac{1}{3}\ln 5$	A1
	Obtain answer y = 1.29 (3s.f. only)	A1
	Total:	7

302. 9709_s17_MS_32 Q: 5

	Answer	Mark
(i)	State $\frac{dy}{dt} = -\frac{2y}{(1+t)^2}$, or equivalent	**B1**
	Separate variables correctly and attempt integration of one side	**M1**
	Obtain term $\ln y$, or equivalent	**A1**
	Obtain term $\frac{2}{(1+t)}$, or equivalent	**A1**
	Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a\ln y$ and $\frac{b}{1+t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{2}{1+t} - 2 + \ln 100$	**A1**
	Total:	**6**
(ii)	State that the mass of B approaches $\frac{100}{e^2}$, or **exact** equivalent	**B1**
	State or imply that the mass of A tends to zero	**B1**
	Total:	**2**

303. 9709_s17_MS_33 Q: 8

	Answer	Mark
(i)	Justify the given differential equation	**B1**
	Total:	**1**
(ii)	Separate variables correctly and attempt to integrate one side	**B1**
	Obtain term kt, or equivalent	**B1**
	Obtain term $-\ln(50-x)$, or equivalent	**B1**
	Evaluate a constant, or use limits $x=0$, $t=0$ in a solution containing terms $a\ln(50-x)$ and bt	**M1***
	Obtain solution $-\ln(50-x)=kt-\ln 50$, or equivalent	**A1**
	Use $x=25$, $t=10$ to determine k	**DM1**
	Obtain correct solution in any form, e.g. $\ln 50-\ln(50-x)=\frac{1}{10}(\ln 2)t$	**A1**
	Obtain answer $x=50(1-\exp(-0.0693t))$, or equivalent	**A1**
	Total:	**8**

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304. 9709_w17_MS_31 Q: 6

	Answer	Mark
	Separate variables correctly and attempt integration of one side	**B1**
	Obtain term $\tan y$, or equivalent	**B1**
	Obtain term of the form $k\ln\cos x$, or equivalent	**M1**
	Obtain term $-4\ln\cos x$, or equivalent	**A1**
	Use $x=0$ and $y=\frac{1}{4}\pi$ in solution containing $a\tan y$ and $b\ln\cos x$ to evaluate a constant, or as limits	**M1**
	Obtain correct solution in any form, e.g. $\tan y=4\ln\sec x+1$	**A1**
	Substitute $y=\frac{1}{3}\pi$ in solution containing terms $a\tan y$ and $b\ln\cos x$, and use correct method to find x	**M1**
	Obtain answer $x=0.587$	**A1**
		8

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305. 9709_w17_MS_32 Q: 5

	Answer	Mark
	Separate variables and obtain $\int \frac{1}{y} \mathrm{d}y = \int \frac{x+2}{x+1} \mathrm{d}x$	**B1**
	Obtain term $\ln y$	**B1**
	Use an appropriate method to integrate $(x+2)/(x+1)$	***M1**
	Obtain integral $x + \ln(x+1)$, or equivalent, e.g. $\ln(x+1) + x + 1$	**A1**
	Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	**DM1**
	Obtain correct solution in x and y in any form e.g. $\ln y = x + \ln(x+1) - 1$	**A1**
	Obtain answer $y = (x+1)\mathrm{e}^{x-1}$	**A1**
		7

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306. 9709_m16_MS_32 Q: 7

	Answer	Mark	
(i)	Separate variables and attempt integration of one side	**M1**	
	Obtain term $-\mathrm{e}^{-y}$	**A1**	
	Integrate $x\mathrm{e}^x$ by parts reaching $x\mathrm{e}^x \pm \int \mathrm{e}^x \,\mathrm{d}x$	**M1**	
	Obtain integral $x\mathrm{e}^x - \mathrm{e}^x$	**A1**	
	Evaluate a constant, or use limits $x = 0, y = 0$	**M1**	
	Obtain correct solution in any form	**A1**	
	Obtain final answer $y = -\ln(\mathrm{e}^x(1-x))$, or equivalent	**A1**	[7]
(ii)	Justify the given statement	**B1**	[1]

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307. 9709_s16_MS_31 Q: 4

Answer	Mark
Separate variables and attempt integration of at least one side	**M1***
Obtain term $\ln y$	**A1**
Obtain terms $\ln x - x^2$	**A1**
Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits	**DM1***
Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$	**A1**
Obtain correct expression for y, free of logarithms, i.e. $y = 2x\exp(1-x^2)$	**A1**
	[6]

308. 9709_s16_MS_32 Q: 6

	Answer	Mark	
(i)	Separate variables correctly and attempt integration of at least one side	**B1**	
	Obtain term $\ln x$	**B1**	
	Obtain term of the form $k\ln(3+\cos 2\theta)$, or equivalent	**M1**	
	Obtain term $-\frac{1}{2}\ln(3+\cos 2\theta)$, or equivalent	**A1**	
	Use $x = 3$, $\theta = \frac{1}{4}\pi$ to evaluate a constant or as limits in a solution with terms $a\ln x$ and $b\ln(3+\cos 2\theta)$, where $ab \neq 0$	**M1**	
	State correct solution in any form, e.g. $\ln x = -\frac{1}{2}\ln(3+\cos 2\theta)+\frac{3}{2}\ln 3$	**A1**	
	Rearrange in a correct form, e.g. $x = \sqrt{\left(\frac{27}{3+\cos 2\theta}\right)}$	**A1**	[7]
(ii)	State answer $x = 3\sqrt{3}/2$, or exact equivalent (accept decimal answer in [2.59, 2.60])	**B1**	[1]

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309. 9709_s16_MS_33 Q: 5

Answer	Mark
Separate variables and make reasonable attempt at integration of either integral	**M1**
Obtain term $\frac{1}{2}e^{2y}$	**B1**
Use Pythagoras	**M1**
Obtain terms $\tan x - x$	**A1**
Evaluate a constant or use $x = 0$, $y = 0$ as limits in a solution containing terms $ae^{\pm 2y}$ and $b\tan x$, ($ab \neq 0$)	**M1**
Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$	**A1**
Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where $a > 0$	**M1**
Obtain answer $y = 0.179$	**A1**
	[8]

310. 9709_w16_MS_31 Q: 10

	Answer	Mark	
(i)	Separate variables correctly and integrate at least one side	**M1**	
	Integrate and obtain term *kt*, or equivalent	**A1**	
	Carry out a relevant method to obtain *A* and *B* such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent	**M1***	
	Obtain $A = B = \frac{1}{4}$, or equivalent	**A1**	
	Integrate and obtain terms $\frac{1}{4}\ln x - \frac{1}{4}\ln(4-x)$, or equivalent	**A1√**	
	EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
	Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$	**A1**	
	Use a second pair of limits and determine *k*	**DM1**	
	Obtain the given exact answer correctly	**A1**	
	OR: Use both pairs of limits in a definite integral	**M1***	
	Obtain the given exact answer correctly	**A1**	
	Substitute *k* and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
	Obtain $\ln\frac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	**A1**	[9]
(ii)	Substitute $x = 3.6$ and solve for t	**M1**	
	Obtain answer $t = 4$	**A1**	[2]

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311. 9709_w16_MS_33 Q: 5

	Answer	Mark	
(i)	State equation $\frac{dy}{dx} = \frac{1}{2}xy$	**B1**	[1]
(ii)	Separate variables correctly and attempts to integrate one side of equation	**M1**	
	Obtain terms of the form $a\ln y$ and bx^2	**A1**	
	Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing $a\ln y$ or bx^2	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$	**A1**	
	Obtain correct expression for *y*, e.g. $y = 2e^{\frac{1}{4}x^2}$	**A1**	[5]
(iii)	Show correct sketch for $x \geqslant 0$. Needs through (0, 2) and rapidly increasing positive gradient.	**B1**	[1]

312. 9709_s15_MS_31 Q: 7

Answer	Mark	
Separate variables and factorise to obtain $\frac{dy}{(3y+1)(y+3)} = 4x\,dx$ or equivalent	B1	
State or imply the form $\frac{A}{3y+1} + \frac{B}{y+3}$ and use a relevant method to find A or B	M1	
Obtain $A = \frac{3}{8}$ and $B = -\frac{1}{8}$	A1	
Integrate to obtain form $k_1\ln(3y+1) + k_2\ln(y+3)$	M1	
Obtain correct $\frac{1}{8}\ln(3y+1) - \frac{1}{8}\ln(y=3) = 2x^2$ or equivalent	A1	
Substitute $x = 0$ and $y = 1$ in equation of form $k_1\ln(3y+1) + k_2\ln(y+3) = k_3x^2 + c$ to find a value of c	M1	
Obtain $c = 0$	A1	
Use correct process to obtain equation without natural logarithm present	M1	
Obtain $y = \frac{3e^{16x^2} - 1}{3 - e^{16x^2}}$ or equivalent	A1	[9]

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313. 9709_s15_MS_32 Q: 9

	Answer	Mark	
(i)	Separate variables correctly and attempt integration of one side	B1	
	Obtain term $\ln x$	B1	
	Obtain term of the form $a\ln(k + e^{-t})$	M1	
	Obtain term $-\ln(k + e^{-t})$	A1	
	Evaluate a constant or use limits $x = 10$, $t = 0$ in a solution containing terms $a\ln(k + e^{-t})$ and $b\ln x$	M1*	
	Obtain correct solution in any form, e.g. $\ln x - \ln 10 = -\ln(k + e^{-t}) + \ln(k + 1)$	A1	**[6]**
(ii)	Substitute $x = 20$, $t = 1$ and solve for k	M1(dep*)	
	Obtain the given answer	A1	**[2]**
(iii)	Using $e^{-t} \to 0$ and the given value of k, find the limiting value of x	M1	
	Justify the given answer	A1	**[2]**

314. 9709_s15_MS_33 Q: 7

	Answer	Mark	
(i)	Separate variables correctly and integrate one side	B1	
	Obtain term $2\sqrt{M}$, or equivalent	B1	
	Obtain term $50k\sin(0.02t)$, or equivalent	B1	
	Evaluate a constant of integration, or use limits $M = 100$, $t = 0$ in a solution with terms of the form $a\sqrt{M}$ and $b\sin(0.02t)$	M1*	
	Obtain correct solution in any form, e.g. $2\sqrt{M} = 50k\sin(0.02t) + 20$	A1	**5**
(ii)	Use values $M = 196$, $t = 50$ and calculate k	M1(dep*)	
	Obtain answer $k = 0.190$	A1	**2**
(iii)	State an expression for M in terms of t, e.g. $M = (4.75\sin(0.02t) + 10)^2$	M1(dep*)	
	State that the least possible number of micro-organisms is 28 or 27.5 or 27.6 (27.5625)	A1	**2**

315. 9709_w15_MS_31 Q: 8

Answer	Mark	
Separate variables and integrate one side	**B1**	
Obtain term $\ln(x+2)$	**B1**	
Use cos 2A formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$	**M1**	
Obtain correct form $(1 - \cos 4\theta)/2$, or equivalent	**A1**	
Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	**A1**√	
Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x+2), d\sin(4\theta), e\theta$	**M1**	
Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	**A1**	
Use correct method for solving an equation of the form $\ln(x+2) = f$	**M1**	
Obtain answer $x = 0.962$	**A1**	[9]

316. 9709_w15_MS_33 Q: 10

	Answer		Mark	
(i)	State $\frac{dN}{dt} = k(N-150)$		**B1**	[1]
(ii)	Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k		**M1**	
	Obtain $k = 0.08$		**A1**	
	Separate variables and obtain general solution involving $\ln(N-150)$		**M1***	
	Obtain $\ln(N-150) = 0.08t + c$ (following their k) or $\ln(N-150) = kt + c$		**A1√**	
	Substitute $t = 0$ and $N = 650$ to find c		**dep M1***	
	Obtain $\ln(N-150) = 0.08t + \ln 500$ or equivalent		**A1**	
	Obtain $N = 500e^{0.08t} + 150$		**A1**	[7]
(iii)	Either	Substitute $t = 15$ to find N or solve for t with $N = 2000$	**M1**	
	Obtain	Either $N = 1810$ or $t = 16.4$ and conclude target not met	**A1**	[2]

317. 9709_s20_MS_31 Q: 10

(a)(i)	Multiply numerator and denominator by $a - 2i$, or equivalent	M1
	Use $i^2 = -1$ at least once	A1
	Obtain answer $\frac{6}{a^2+4} + \frac{3ai}{a^2+4}$	A1
		3
(a)(ii)	Either state that arg $u = -\frac{1}{3}\pi$ or express u^* in terms of a (FT on u)	B1
	Use correct method to form an equation in a	M1
	Obtain answer $a = -2\sqrt{3}$	A1
		3
(b)(i)	Show the perpendicular bisector of points representing 2i and 1 + i	B1
	Show the point representing 2 + i	B1
	Show a circle with radius 2 and centre 2 + i (FT on the position of the point for 2 + i)	B1FT
	Shade the correct region	B1
		4
(b)(ii)	State or imply the critical point 2 + 3i	B1
	Obtain answer 56.3° or 0.983 radians	B1
		2

318. 9709_s20_MS_32 Q: 8

(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $x - y = 3$ and $3x + y = 5$	**A1**
	Solve and obtain answer $z = 2 - i$	**A1**
		4
(b)(i)	Show a point representing $2 + 2i$	**B1**
	Show a circle with radius 1 and centre not at the origin (**FT** is on the point representing the centre)	**B1 FT**
	Show the correct half line from 4i	**B1**
	Shade the correct region	**B1**
		4
(b)(ii)	Carry out a complete method for finding the least value of Im z	**M1**
	Obtain answer $2 - \frac{1}{2}\sqrt{2}$, or exact equivalent	**A1**
		2

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319. 9709_s20_MS_33 Q: 9

(a)	Eliminate u or w and obtain an equation w or u	**M1**
	Obtain a quadratic in u or w, e.g. $u^2 - 2iu - 6 = 0$ or $w^2 + 2iw - 6 = 0$	**A1**
	Solve a 3-term quadratic for u or for w	**M1**
	Obtain answer $u = \sqrt{5} + i$, $w = \sqrt{5} - i$	**A1**
	Obtain answer $u = -\sqrt{5} + i$, $w = -\sqrt{5} - i$	**A1**
		5
(b)	Show the point representing $2 + 2i$	**B1**
	Show a circle with centre $2 + 2i$ and radius 2 (**FT** is on the position of $2 + 2i$)	**B1 FT**
	Show half-line from origin at 45° to the positive x-axis	**B1**
	Show line for Re $z = 3$	**B1**
	Shade the correct region	**B1**
		5

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320. 9709_w20_MS_31 Q: 2

	Answer	**Mark**	**Partial Marks**
	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing $1 - i$	**B1**	
	Show a circle with centre $1 - i$ and radius 1	**B1 FT**	The FT is on the position of $1 - i$.
	Shade the appropriate region	**B1 FT**	The FT is on the position of $1 - i$. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm i$ and radius 1
		4	

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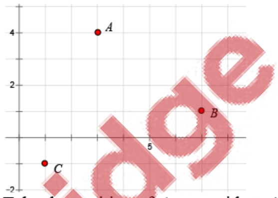
321. 9709_w20_MS_31 Q: 7

	Answer	Mark	Partial Marks
(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	M1	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2=-1$ correctly at least once	M1	$1-5$ or $4+10$ seen
	Complete the verification correctly	A1	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
(b)	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	M1	
	Obtain x^2+2x+6	A1	
	Obtain root $x=\frac{3}{2}$	A1	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	$(x+1-\sqrt{5}\mathrm{i})(x+1+\sqrt{5}\mathrm{i})(2x+a)=2x^3+x^2+6x-18$	M1	
	$(1-\sqrt{5}\,\mathrm{i})(1+\sqrt{5}\,\mathrm{i})\,a=-18$	A1	
	$6a=-18$ $a=-3$ leading to $x=\frac{3}{2}$	A1	OE
(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	POR = 6 SOR = − 2	M1	
	Obtain x^2+2x+6	A1	
	Obtain root $x=\frac{3}{2}$	A1	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	POR $\left(-1-\sqrt{5}\mathrm{i}\right)\left(-1+\sqrt{5}\mathrm{i}\right)a=9$	M1 A1	
	Obtain root $x=\frac{3}{2}$	A1	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	B1	
	SOR $\left(-1-\sqrt{5}\mathrm{i}\right)+\left(-1+\sqrt{5}\mathrm{i}\right)+a=-\frac{1}{2}$	M1 A1	
	Obtain root $x=\frac{3}{2}$	A1	OE
		4	

322. 9709_w20_MS_32 Q: 6

	Answer	Mark	Partial Marks
(a)	Multiply numerator and denominator by 1 + i, or equivalent	**M1**	Must multiply out
	Obtain numerator 6 + 8i or denominator 2	**A1**	
	Obtain final answer $u = 3 + 4\mathrm{i}$	**A1**	
	Alternative method for question 6(a)		
	Multiply out $(1-i)(x+iy)=7+i$ and compare real and imaginary parts	**M1**	
	Obtain $x+y=7$ or $y-x=1$	**A1**	
	Obtain final answer $u = 3 + 4\mathrm{i}$	**A1**	
		3	
(b)	Show the point A representing u in a relatively correct position	**B1 FT**	The FT is on $xy \neq 0$.
	Show the other two points B and C in relatively correct positions: approximately equal distance above / below real axis	**B1**	 Take the position of A as a guide to 'scale' if axes not marked
		2	
(c)	State or imply $\arg(1 - \mathrm{i}) = -\frac{1}{4}\pi$	**B1**	Arg$C$
	Substitute exact arguments in $\arg(7 + \mathrm{i}) - \arg(1 - \mathrm{i}) = \arg u$	**M1**	Must see a statement about the relationship between the Args e.g. Arg$A$ = Arg$B$ – Arg$C$ or equivalent exact method
	Obtain $\tan^{-1}\left(\frac{4}{3}\right)=\tan^{-1}\left(\frac{1}{7}\right)+\frac{1}{4}\pi$ correctly	**A1**	Obtain given answer correctly from *their* $u=k(3+4i)$
		3	

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323. 9709_m19_MS_32 Q: 7

	Answer	Mark	Partial Marks
(a)	Use quadratic formula to solve for z	**M1**	
	Use $\mathrm{i}^2 = -1$ throughout	**M1**	
	Obtain correct answer in any form	**A1**	
	Multiply numerator and denominator by 1 – i, or equivalent	**M1**	
	Obtain final answer, e.g. 1 – i	**A1**	
	Obtain second final answer, e.g. $\frac{5}{2}+\frac{1}{2}\mathrm{i}$	**A1**	
		6	
(b)	Show the point representing u in relatively correct position	**B1**	
	Show the horizontal line through $z = \mathrm{i}$	**B1**	
	Show correct half-lines from u, one of gradient 1 and the other vertical	**B1ft**	
	Shade the correct region	**B1**	
		4	

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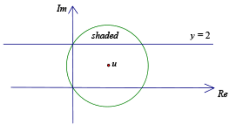
324. 9709_s19_MS_31 Q: 10

	Answer	Mark	Partial Marks
(i)	State or imply $r = 2$	**B1**	Accept $\sqrt{4}$
	State or imply $\theta = \frac{1}{6}\pi$	**B1**	
	Use a correct method for finding the modulus or the argument of u^4	**M1**	Allow correct answers from correct u with minimal working shown
	Obtain modulus 16	**A1**	
	Obtain argument $\frac{2}{3}\pi$	**A1**	Accept $16e^{i\frac{2\pi}{3}}$
		5	
(ii)	Substitute u and carry out a correct method for finding u^3	**M1**	$\left(u^3 = 8i\right)$ Follow *their* u^3 if found in part **(i)**
	Verify u is a root of the given equation	**A1**	
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Alternative method		
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Form quadratic factor and divide cubic by quadratic	**M1**	$\left(z - \sqrt{3} - i\right)\left(z - \sqrt{3} + i\right)\left(= z^2 - 2\sqrt{3}z + 4\right)$
	Verify that remainder is zero and hence that u is a root of the given equation	**A1**	
		3	
(iii)	Show the point representing u in a relatively correct position	**B1**	
	Show a circle with centre u and radius 2	**B1**	**FT** on the point representing u. Condone near miss of origin
	Show the line $y = 2$	**B1**	
	Shade the correct region	**B1**	
	Show that the line and circle intersect on $x = 0$	**B1**	Condone near miss
		5	

325. 9709_s19_MS_32 Q: 5

	Answer	Mark	Partial Marks
(i)	State answer $-1-\sqrt{3}\mathrm{i}$	B1	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	
(ii)	Substitute $x = -1+\sqrt{3}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $k = 2$	A1	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$	M1	Could use factor theorem from this point. Need to see working. M1 for correct testing of correct root or allow M1 for three unsuccessful valid attempts.
	Obtain x^2+2x+4	A1	Using factor theorem, obtain $\mathrm{f}\left(-\frac{1}{2}\right)=0$
	Obtain root $x = -\frac{1}{2}$, or equivalent, *via* division or inspection	A1	Final answer
(ii)	**Alternative method 1**		
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$ (multiplying two linear factors or using sum and product of roots)	M1	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain x^2+2x+4	A1	Allow M1A0 for x^2+2x+3
	Obtain linear factor $kx+1$ and compare coefficients of x or x^2 and solve for k	M1	Can find the factor by inspection or by long division Must get to zero remainder
	Obtain $k = 2$	A1	
	Obtain root $x = -\frac{1}{2}$	A1	Final answer
			Note: Verification that $x = -\frac{1}{2}$ is a root is worth no marks without a clear demonstration of how the root was obtained
(ii)	**Alternative method 2**		
	Use equation for sum of roots of cubic and use equation for product of roots of cubic	M1	
	Use $\mathrm{i}^2 = -1$ correctly at least once	M1	Allow for relevant use at any point in the solution
	Obtain $-\frac{5}{k} = -2+\gamma$, $\quad -\frac{4}{k} = 4\gamma$	A1	
	Solve simultaneous equations for k and γ	M1	
	Obtain $k = 2$	A1	
	Obtain root $\gamma = -\frac{1}{2}$	A1	Final answer
		6	

326. 9709_s19_MS_33 Q: 8

	Answer	Mark	Partial Marks
(i)	Multiply numerator and denominator by $1+\sqrt{3}\mathrm{i}$, or equivalent	M1	
	$4\mathrm{i}-4\sqrt{3}$ and $3+1$	A1	
	Obtain final answer $-\sqrt{3}+\mathrm{i}$	A1	
		3	
(ii)	State that the modulus of u is 2	B1	
	State that the argument of u is $\frac{5}{6}\pi$ (or 150°)	B1	
		2	
(iii)	Show a circle with centre the origin and radius 2	B1	
	Show u in a relatively correct position	B1	FT
	Show the perpendicular bisector of the line joining u and the origin	B1	FT
	Shade the correct region	B1	
		4	

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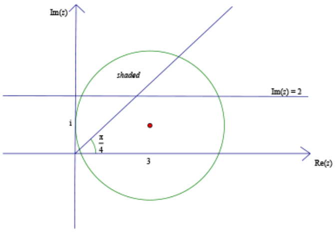
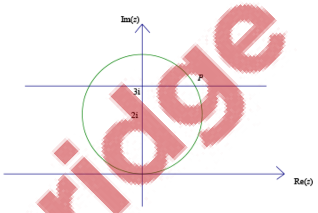
327. 9709_w19_MS_31 Q: 10

	Answer	Mark	Partial Marks
(a)	Square $a+\mathrm{i}b$ and equate real and imaginary parts to -3 and $-2\sqrt{10}$ respectively	*M1	
	Obtain $a^2-b^2=-3$ and $2ab=-2\sqrt{10}$	A1	
	Eliminate one unknown and find an equation in the other	DM1	
	Obtain $a^4+3a^2-10=0$, or $b^4-3b^2-10=0$, or horizontal 3-term equivalent	A1	
	Obtain answers $\pm\left(\sqrt{2}-\sqrt{5}\mathrm{i}\right)$, or exact equivalent	A1	
		5	
(b)	Show point representing $3+\mathrm{i}$ in relatively correct position	B1	
	Show a circle with radius 3 and centre not at the origin	B1	
	Show correct half line from the origin at $\frac{1}{4}\pi$ to the real axis	B1	
	Show horizontal line $y=2$	B1	
	Shade the correct region	B1	
		5	

328. 9709_w19_MS_32 Q: 7

	Answer	Mark	Partial Marks
(a)	Substitute and obtain a correct horizontal equation in x and y in any form	**B1**	$zz^* + \mathrm{i}z - 2z^* = 0 \Rightarrow$ $x^2 + y^2 + \mathrm{i}x - y - 2x + 2\mathrm{i}y = 0$ Allow if still includes brackets and/or i^2
	Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to zero OE	***M1**	For their horizontal equation
	Obtain two correct equations e.g. $x^2 + y^2 - y - 2x = 0$ and $x + 2y = 0$	**A1**	Allow $\mathrm{i}x + 2\mathrm{i}y = 0$
	Solve for x or for y	**DM1**	
	Obtain answer $\frac{6}{5} - \frac{3}{5}\mathrm{i}$ and no other	**A1**	OE, condone $\frac{1}{5}(6 - 3\mathrm{i})$
		5	
(b)(i)	Show a circle with centre 2i and radius 2	**B1**	
	Show horizontal line $y = 3$ – in first and second quadrant	**B1**	
			SC: For clearly labelled axes not in the conventional directions, allow B1 for a fully 'correct' diagram.
		2	
(b)(ii)	Carry out a complete method for finding the argument. (Not by measuring the sketch)	**M1**	$\left(z = \sqrt{3} + 3\mathrm{i}\right)$ Must show working if using 1.7 in place of $\sqrt{3}$.
	Obtain answer $\frac{1}{3}\pi$ (or $60°$)	**A1**	**SC:** Allow B2 for 60° with no working
		2	

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329. 9709_w19_MS_33 Q: 6

	Answer	Mark	Partial Marks
(i)	Obtain answer $w = \frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$	**B1**	
		1	
(ii)	Show point representing u	**B1**	
	Show point representing v in relatively correct position	**B1**	
		2	
(iii)	Explain why the moduli are equal	**B1**	
	Explain why the arguments are equal	**B1**	
	Use $\mathrm{i}^2 = -1$ and obtain $2uw$ in the given form	**M1**	
	Obtain answer $1 - 2\sqrt{3} + \left(2 + \sqrt{3}\right)\mathrm{i}$	**A1**	
		4	

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330. 9709_m18_MS_32 Q: 9

	Answer	Mark
(i)(a)	Substitute $x = 1 + 2\text{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**
	Use $\text{i}^2 = -1$ correctly at least once and solve for k	**M1**
	Obtain answer $k = 15$	**A1**
		3
(i)(b)	State answer $1 - 2\text{i}$	**B1**
	Carry out a complete method for finding a quadratic factor with zeros $1 + 2\text{i}$ and $1 - 2\text{i}$	**M1**
	Obtain $x^2 - 2x + 5$	**A1**
	Obtain root $-\frac{3}{2}$, or equivalent, *via* division or inspection	**A1**
		4
(ii)	Show a circle with centre $1 + 2\text{i}$	**B1**
	Show a circle with radius 1	**B1**
	Carry out a complete method for calculating the least value of arg z	**M1**
	Obtain answer 0.64	**A1**
		4

331. 9709_s18_MS_31 Q: 7

	Answer	Mark
(i)	Use quadratic formula, or completing the square, or the substitution $z = x + \mathrm{i}y$ to find a root, using $\mathrm{i}^2 = -1$	**M1**
	Obtain a root, e.g. $-\sqrt{6} - \sqrt{2}\mathrm{i}$	**A1**
	Obtain the other root, e.g. $-\sqrt{6} - \sqrt{2}\mathrm{i}$	**A1**
		3
(ii)	Represent both roots in relatively correct positions	**B1ft**
		1
(iii)	State or imply correct value of a relevant length or angle, e.g. *OA*, *OB*, *AB*, angle between *OA* or *OB* and the real axis	**B1ft**
	Carry out a complete method for finding angle *OAB*	**M1**
	Obtain *AOB* = 60° correctly	**A1**
		3
(iv)	Give a complete justification of the given statement	**B1**
		1

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332. 9709_s18_MS_32 Q: 7

	Answer	Mark	Partial Marks
(i)	Substitute in *uv*, expand the product and use $\mathrm{i}^2 = -1$	**M1**	
	Obtain answer $uv = -11 - 5\sqrt{3}\mathrm{i}$	**A1**	
	EITHER: Substitute in *u/v* and multiply numerator and denominator by the conjugate of *v*, or equivalent	**M1**	
	Obtain numerator $-7 + 7\sqrt{3}\mathrm{i}$ or denominator 7	**A1**	
	Obtain final answer $-1 + \sqrt{3}\mathrm{i}$	**A1**	
	OR: Substitute in *u/v* , equate to $x + \mathrm{i}y$ and solve for *x* or for *y*	**M1**	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
	Obtain $x = -1$ or $y = \sqrt{3}$	**A1**	
	Obtain final answer $-1 + \sqrt{3}\,\mathrm{i}$	**A1**	
		5	

	Answer	Mark	Partial Marks
(ii)	Show the points *A* and *B* representing *u* and *v* in relatively correct positions	**B1**	
	Carry out a complete method for finding angle *AOB*, e.g. calculate arg(*u*/*v*) If using $\theta = \tan^{-1}\left(-\sqrt{3}\right)$ must refer to $\arg\left(\frac{u}{v}\right)$	**M1**	*OR*: $\tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan(a-b) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1-\frac{2}{9}}$ $= -\sqrt{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{\begin{pmatrix}-3\sqrt{3}\\1\end{pmatrix}\begin{pmatrix}\sqrt{3}\\2\end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	**A1**	Given answer so check working carefully
		3	

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333. 9709_s18_MS_33 Q: 9

	Answer	Mark
(a)	Substitute and obtain a correct equation in *x* and *y*	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in *x* and *y*, e.g. $3x - y = 1$ and $3y - x = 5$	**A1**
	Solve and obtain answer $z = 1 + \mathbf{2(i)}$	**A1**
	Total:	**4**
(b)	Show a circle with radius 3	**B1**
	Show the line $y = 2$ extending in both quadrants	**B1**
	Shade the correct region	**B1**
	Carry out a complete method for finding the greatest value of arg *z*	**M1**
	Obtain answer 2.41	**A1**
	Total:	**5**

334. 9709_w18_MS_31 Q: 8

	Answer	Mark	Partial Marks
(i)	*EITHER*: Multiply numerator and denominator by 1 + 2i, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y	**M1**	
	Obtain quotient $-\frac{4}{5}+\frac{7}{5}$i, or equivalent	**A1**	
	Use correct method to find either r or θ	**M1**	
	Obtain $r = 1.61$	**A1**	
	Obtain $\theta = 2.09$	**A1**	
	OR: Find modulus or argument of 2 + 3i or of 1 – 2i	**B1**	
	Use correct method to find r	**M1**	
	Obtain $r = 1.61$	**A1**	
	Use correct method to find θ	**M1**	
	Obtain $\theta = 2.09$	**A1**	
		5	
(ii)	Show a circle with centre 3 – 2i	**B1**	
	Show a circle with radius 1	**B1ft**	Centre not at the origin
	Carry out a correct method for finding the least value of $\|z\|$	**M1**	
	Obtain answer $\sqrt{13}\ -1$	**A1**	
		4	

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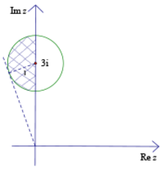
335. 9709_w18_MS_32 Q: 9

	Answer	Mark	Partial Marks
(a)(i)	Multiply numerator and denominator by 1 + 2i, or equivalent	**M1**	Requires at least one of $2+10\mathrm{i}+12\mathrm{i}^2$ and $1-4\mathrm{i}^2$ together with use of $\mathrm{i}^2=-1$. Can be implied by $\frac{-10+10\mathrm{i}}{5}$
	Obtain quotient – 2 + 2i	**A1**	
	Alternative		
	Equate to x + iy, obtain two equations in x and y and solve for x or for y	**M1**	$x+2y=2,\quad y-2x=6$
	Obtain quotient – 2 + 2i	**A1**	
		2	
(a)(ii)	Use correct method to find either r or θ	**M1**	If only finding θ, need to be looking for θ in the correct quadrant
	Obtain $r=2\sqrt{2}$, or exact equivalent	**A1ft**	ft their x + iy
	Obtain $\theta=\frac{3}{4}\pi$ from exact work	**A1ft**	ft on k(-1 + i) for k > 0 Do not ISW
		3	

	Answer	Mark	Partial Marks
(b)	Show a circle with centre 3i	**B1**	
	Show a circle with radius 1	**B1ft**	Follow through their centre provided not at the origin For clearly unequal scales, should be an ellipse
	All correct with even scales and **shade** the correct region	**B1**	
	Carry out a correct method for calculating greatest value of arg z	**M1**	e.g. $\arg z = \frac{\pi}{2} + \sin^{-1}\frac{1}{3}$
	Obtain answer 1.91	**A1**	
		5	

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336. 9709_m17_MS_32 Q: 8

	Answer	Mark
(i)	Substitute $z = -1 + \mathrm{i}$ and attempt expansions of the z^2 and z^4 terms	**M1**
	Use $\mathrm{i}^2 = -1$ at least once	**M1**
	Complete the verification correctly	**A1**
	Total:	**3**
(ii)	State second root $z = -1 - \mathrm{i}$	**B1**
	Carry out a complete method for finding a quadratic factor with zeros $-1 + \mathrm{i}$ and $-1 - \mathrm{i}$	**M1**
	Obtain $z^2 + 2z + 2$, or equivalent	**A1**
	Attempt division of p(z) by $z^2 + 2z + 2$ and reach a partial quotient $z^2 + kz$	**M1**
	Obtain quadratic factor $z^2 - 2z + 5$	**A1**
	Solve 3-term quadratic and use $\mathrm{i}^2 = -1$	**M1**
	Obtain roots $1 + 2\mathrm{i}$ and $1 - 2\mathrm{i}$	**A1**
	Total:	**7**

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337. 9709_s17_MS_31 Q: 7

	Answer	Mark
(i)	State that $u - 2w = -7 - i$	**B1**
	EITHER: Multiply numerator and denominator of $\frac{u}{w}$ by 3 – 4i, or equivalent	**(M1**
	Simplify the numerator to 25 + 25i or denominator to 25	**A1**
	Obtain final answer 1 + i	**A1)**
	OR: Obtain two equations in x and y and solve for x or for y	**(M1**
	Obtain $x = 1$ or $y = 1$	**A1**
	Obtain final answer 1 + i	**A1)**
	Total:	**4**
(ii)	Find the argument of $\frac{u}{w}$	**M1**
	Obtain the given answer	**A1**
	Total:	**2**
(iii)	State that OB and CA are parallel	**B1**
	State that $CA = 2OB$, or equivalent	**B1**
	Total:	**2**

338. 9709_s17_MS_32 Q: 6

	Answer	Mark
(i)	*EITHER:* Substitute $x = 2 - \mathrm{i}$ (or $x = 2 + \mathrm{i}$) in the equation and attempt expansions of x^2 and x^3	(M1
	Equate real and/or imaginary parts to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*1: Substitute $x = 2 - \mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	(M1
	Substitute $x = 2 + \mathrm{i}$ in the equation and add/subtract the two equations	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*2: Factorise to obtain $(x-2+\mathrm{i})(x-2-\mathrm{i})(x-p)$ $\left(=\left(x^2-4x+5\right)(x-p)\right)$	(M1
	Compare coefficients	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*3: Obtain the quadratic factor (x^2-4x+5)	(M1
	Use algebraic division to obtain a real linear factor of the form $x - p$ and set the remainder equal to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*4: Use $\alpha\beta = 5$ and $\alpha + \beta = 4$ in $\alpha\beta + \beta\gamma + \gamma\alpha = -3$	(M1
	Solve for γ and use in $\alpha\beta\gamma = -b$ and/or $\alpha + \beta + \gamma = -a$	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)

	Answer	Mark
	OR5: Factorise as (x–- (2-i))($x^2 + ex + g$) and compare coefficients to form an equation in a and b	**(M1**
	Equate real and/or imaginary parts to zero	**M1**
	Obtain $a = -2$	**A1**
	Obtain $b = 10$	**A1)**
	Total:	**4**
(ii)	Show a circle with centre 2 – i in a relatively correct position	**B1**
	Show a circle with radius 1 and centre not at the origin	**B1**
	Show the perpendicular bisector of the line segment joining 0 to – i	**B1**
	Shade the correct region	**B1**
	Total:	**4**

339. 9709_s17_MS_33 Q: 11

	Answer	Mark
(a)	Solve for z or for w	**M1**
	Use $i^2 = -1$	**M1**
	Obtain $w = \frac{i}{2-i}$ or $z = \frac{2+i}{2-i}$	**A1**
	Multiply numerator and denominator by the conjugate of the denominator	**M1**
	Obtain $w = -\frac{1}{5} + \frac{2}{5}i$	**A1**
	Obtain $z = \frac{3}{5} + \frac{4}{5}i$	**A1**
	Total:	**6**
(b)	*EITHER:* Find $\pm\left[2+\left(2-2\sqrt{3}\right)i\right]$	**(B1**
	Multiply by 2i (or –2i)	**M1***
	Add result to v	**DM1**
	Obtain answer $4\sqrt{3}-1+6i$	**A1)**
	OR: State $\frac{z-v}{v-u} = ki$, or equivalent	**(M1**
	State $k = 2$	**A1**
	Substitute and solve for z even if i omitted	**M1**
	Obtain answer $4\sqrt{3}-1+6i$	**A1)**
	Total:	**4**

340. 9709_w17_MS_31 Q: 7

	Answer	Mark
(a)	Square x + iy and equate real and imaginary parts to 8 and –15	**M1**
	Obtain $x^2 - y^2 = 8$ and $2xy = -15$	**A1**
	Eliminate one unknown and find a horizontal equation in the other	**M1**
	Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent	**A1**
	Obtain answers $\pm\frac{1}{\sqrt{2}}(5-3\text{i})$ or equivalent	**A1**
		5
(b)	Show a circle with centre $2+\text{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 and centre not at the origin	**B1**
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	**B1**
	Shade the correct region	**B1**
		4

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341. 9709_w17_MS_32 Q: 7

	Answer	Mark
(i)	State modulus 2	**B1**
	State argument $-\frac{1}{3}\pi$ or $-60°$ ($\frac{5}{3}\pi$ or 300°)	**B1**
		2
(ii)	*EITHER:* Expand $(1-(\sqrt{3})\text{i})^3$ completely and process i^2 and i^3	**(M1**
	Verify that the given relation is satisfied	**A1)**
	OR: $u^3 = 2^3\left(\cos(-\pi) + i\sin(-\pi)\right)$ or equivalent: follow their answers to **(i)**	**(M1**
	Verify that the given relation is satisfied	**A1)**
		2

	Answer	Mark
(iii)	Show a circle with centre $1-(\sqrt{3})\mathrm{i}$ in a relatively correct position	B1
	Show a circle with radius 2 passing through the origin	B1
	Show the line Re $z = 2$	B1
	Shade the correct region	B1
		4

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342. 9709_m16_MS_32 Q: 10

		Answer	Mark	
(a)		Substitute and obtain a correct equation in x and y	**B1**	
		Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts	**M1**	
		Obtain two correct equations, e.g. $x + 2y + 1 = 0$ and $y + 2x = 0$	**A1**	
		Solve for x or for y	**M1**	
		Obtain answer $z = \frac{1}{3} - \frac{2}{3}\mathrm{i}$	**A1**	[5]
(b)	**(i)**	Show a circle with centre $-1+3\mathrm{i}$	**B1**	
		Show a circle with radius 1	**B1**	
		Show the line Im $z = 3$	**B1**	
		Shade the correct region	**B1**	[4]
	(ii)	Carry out a complete method to calculate the relevant angle	**M1**	
		Obtain answer 0.588 radians (accept 33.7°)	**A1**	[2]

343. 9709_s16_MS_31 Q: 10

Answer	Mark
(a) Square x+ iy and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively	**M1**
Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$	**A1**
Eliminate one variable and find an equation in the other	**M1**
Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0$, or 3-term equivalent	**A1**
Obtain answers $\pm(3 - \mathrm{i}\sqrt{2})$	**A1**
	[5]
(b) **(i)** Show point representing 1 + 2i	**B1**
Show circle with radius 1 and centre 1 + 2i	**B1**√
Show a half line from the point representing 1	**B1**
Show line making the correct angle with the real axis	**B1**
	[4]
(ii) State or imply the relevance of the perpendicular from 1 + 2i to the line	**M1**
Obtain answer $\sqrt{2} - 1$ (or 0.414)	**A1**
	[2]

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344. 9709_s16_MS_32 Q: 10

Answer	Mark	
(a) *EITHER*: Use quadratic formula to solve for z	**M1**	
Use $\mathrm{i}^2 = -1$	**M1**	
Obtain a correct answer in any form, simplified as far as $(-2 \pm \mathrm{i}\sqrt{8})/2\mathrm{i}$	**A1**	
Multiply numerator and denominator by i, or equivalent	**M1**	
Obtain final answers $\sqrt{2} + \mathrm{i}$ and $-\sqrt{2} + \mathrm{i}$	**A1**	
OR: Substitute x + iy and equate real and imaginary parts to zero	**M1**	
Use $\mathrm{i}^2 = -1$	**M1**	
Obtain $-2xy + 2x = 0$ and $x^2 - y^2 + 2y - 3 = 0$, or equivalent	**A1**	
Solve for x and y	**M1**	
Obtain final answers $\sqrt{2} + \mathrm{i}$ and $-\sqrt{2} + \mathrm{i}$	**A1**	[5]
(b) **(i)** *EITHER*: Show the point representing 4 + 3i in relatively correct position	**B1**	
Show the perpendicular bisector of the line segment joining this point to the origin	**B1**√	[2]
OR: Obtain correct Cartesian equation of the locus in any form, e.g. $8x + 6y = 25$	**B1**	
Show this line [This f.t. is dependent on using a correct method to determine the equation.]	**B1**√	
(ii) State or imply the relevant point is represented by 2 + 1.5i or is at (2, 1.5)	**B1**	
Obtain modulus 2.5	**B1**√	
Obtain argument 0.64 (or 36.9°) (allow decimals in [0.64, 0.65] or [36.8, 36.9])	**B1**√	[3]

345. 9709_s16_MS_33 Q: 9

	Answer	Mark
(i)	*EITHER*: Multiply numerator and denominator of $\frac{u}{v}$ by 2 + i, or equivalent	**M1**
	Simplify the numerator to –5 +5i or denominator to 5	**A1**
	Obtain final answer –1 + I	**A1**
	OR: Obtain two equations in x and y and solve for x or for y	**(M1**
	Obtain $x = -1$ or $y = 1$	**A1**
	Obtain final answer –1 + I	**A1)**
		[3]
(ii)	Obtain $u + v = 1 + 2i$	**B1**
	In an Argand diagram show points A, B, C representing u, v and $u + v$ respectively	**B1**√
	State that OB and AC are parallel	**B1**
	State that $OB = AC$	**B1**
		[4]
(iii)	Carry out an appropriate method for finding angle AOB, e.g. find $\arg(u / v)$	**M1**
	Show sufficient working to justify the given answer $\frac{3}{4}\pi$	**A1**
		[2]

346. 9709_w16_MS_31 Q: 9

	Answer	Mark	
(a)	EITHER: Use quadratic formula to solve for w	**M1**	
	Use $i^2 = -1$	**M1**	
	Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$	**A1**	
	Multiply numerator and denominator of an answer by –2i + 1, or equivalent	**M1**	
	Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and –1 + 2i	**A1**	
	OR1: Multiply the equation by 1 – 2i	**M1**	
	Use $i^2 = -1$	**M1**	
	Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	**A1**	
	Use quadratic formula or factorise to solve for w	**M1**	
	Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and –1 + 2i	**A1**	
	OR2: Substitute $w = x + iy$ and form equations for real and imaginary parts	**M1**	
	Use $i^2 = -1$	**M1**	
	Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	**A1**	
	Form equation in x only or y only and solve	**M1**	
	Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and –1 + 2i	**A1**	[5]
(b)	Show a circle with centre 1 + i	**B1**	
	Show a circle with radius 2	**B1**	
	Show half-line $\arg z = \frac{1}{4}\pi$	**B1**	
	Show half-line $\arg z = -\frac{1}{4}\pi$	**B1**	
	Shade the correct region	**B1**	[5]

347. 9709_w16_MS_33 Q: 7

	Answer	Mark	
(i)	State modulus $2\sqrt{2}$, or equivalent State argument $-\frac{1}{3}\pi$ (or $-60°$)	**B1** **B1**	[2]
(ii) (a)	State answer $3\sqrt{2}+\sqrt{6}\,\mathrm{i}$	**B1**	
(b)	*EITHER:* Substitute for z and multiply numerator and denominator by conjugate of iz	**M1**	
	Simplify the numerator to $4\sqrt{3}+4\,\mathrm{i}$ or the denominator to 8	**A1**	
	Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$	**A1**	
	OR: Substitute for z, obtain two equations in x and y and solve for x or for y	**M1**	
	Obtain $x=\frac{1}{2}\sqrt{3}$ or $y=\frac{1}{2}$	**A1**	
	Obtain final answer $\frac{1}{2}\sqrt{3}+\frac{1}{2}\mathrm{i}$	**A1**	[4]
(iii)	Show points A and B in relatively correct positions	**B1**	
	Carry out a complete method for finding angle AOB, e.g. calculate the argument of $\dfrac{z^*}{\mathrm{i}z}$	**M1**	
	Obtain the given answer	**A1**	[3]

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348. 9709_s15_MS_31 Q: 8

		Answer	Mark	
(i)	Either	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
		Multiply by $\dfrac{3+4\mathrm{i}}{3+4\mathrm{i}}$ and simplify to $x+\mathrm{i}y$ form or equivalent	M1	
		Confirm given answer $2+4\mathrm{i}$	A1	
	Or	Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
		Obtain two equations in x and y and solve for x or y	M1	
		Confirm given answer $2+4\mathrm{i}$	A1	[3]
(ii)	Identify $4+4$ or $-4+4\mathrm{i}$ as point at either end or state $p=2$ or state $p=-6$		B1	
	Use appropriate method to find both critical values of p		M1	
	State $-6 \leqslant p \leqslant 2$		A1	[3]
(iii)	Identify equation as of form $\lvert z-a\rvert = a$ or equivalent		M1	
	Form correct equation for a not involving modulus, e.g. $(a-2)^2+4^2=a^2$		A1	
	State $\lvert z-5\rvert = 5$		A1	[3]

349. 9709_s15_MS_32 Q: 7

	Answer	Mark	
(i)	Square $x + \mathrm{i}y$ and equate real and imaginary parts to -1 and $4\sqrt{3}$	M1	
	Obtain $x^2 - y^2 = -1$ and $2xy = 4\sqrt{3}$	A1	
	Eliminate one unknown and find an equation in the other	M1	
	Obtain $x^4 + x^2 - 12 = 0$ or $y^4 - y^2 - 12 = 0$, or three term equivalent	A1	
	Obtain answers $\pm(\sqrt{3} + 2\mathrm{i})$	A1	**[5]**
	[If the equations are solved by inspection, give B2 for the answers and B1 for justifying them]		
(ii)	Show a circle with centre $-1 + 4\sqrt{3}$ in a relatively correct position	B1	
	Show a circle with radius 1 and centre not at the origin	B1	
	Carry out a complete method for calculating the greatest value of arg z	M1	
	Obtain answer 1.86 or 106.4°	A1	**[4]**

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350. 9709_s15_MS_33 Q: 8

		Answer	Mark	
(i)	*EITHER*:	Substitute for u in $\frac{\mathrm{i}}{u}$ and multiply numerator and denominator by 1 + i	M1	
		Obtain final answer $-\frac{1}{2} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	
	OR:	Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
		Obtain final answer $-\frac{1}{2} + \frac{1}{2}\mathrm{i}$, or equivalent	A1	**2**
(ii)	Show a point representing u in a relatively correct position		B1	
	Show the bisector of the line segment joining u to the origin		B1	
	Show a circle with centre at the point representing i		B1	
	Show a circle with radius 2		B1	**4**
(iii)	State argument $-\frac{1}{2}\pi$, or equivalent, e.g. 270°		B1	
	State or imply the intersection in the first quadrant represents 2 + i		B1	
	State argument 0.464, (0.4636)or equivalent, e.g. 26.6° (26.5625)		B1	**3**

351. 9709_w15_MS_31 Q: 9

	Answer	Mark	
(i)	Show u in a relatively correct position	**B1**	
	Show u^* in a relatively correct position	**B1**	
	Show $u^* - u$ in a relatively correct position	**B1**	
	State or imply that *OABC* is a parallelogram	**B1**	[4]
(ii)	*EITHER*: Substitute for u and multiply numerator and denominator by 3 + i, or equivalent	**M1**	
	Simplify the numerator to 8 + 6i or the denominator to 10	**A1**	
	Obtain final answer $\frac{4}{5}+\frac{3}{5}\mathrm{i}$, or equivalent	**A1**	
	OR: Substitute for u, obtain two equations in x and y and solve for x or for y	**M1**	
	Obtain $x=\frac{4}{5}$ or $y=\frac{3}{5}$, or equivalent	**A1**	
	Obtain final answer $\frac{4}{5}+\frac{3}{5}\mathrm{i}$, or equivalent	**A1**	[3]
(iii)	State or imply $\arg(u^*/u)=\tan^{-1}(\frac{3}{4})$	**B1**	
	Substitute exact arguments in $\arg(u^*/u)=\arg u^* - \arg u$	**M1**	
	Fully justify the given statement using exact values	**A1**	[3]

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352. 9709_w15_MS_33 Q: 9

		Answer	Mark
(a)	Either	Find w using conjugate of $1+3\mathrm{i}$	**M1**
		Obtain $\dfrac{7-\mathrm{i}}{5}$ or equivalent	**A1**
		Square $x+\mathrm{i}y$ form to find w^2	**M1**
		Obtain $w^2=\dfrac{48-14\mathrm{i}}{25}$ and confirm modulus is 2	**A1**
		Use correct process for finding argument of w^2	**M1**
		Obtain –0.284 radians or $-16.3°$	**A1**
	Or 1	Find w using conjugate of $1+3\mathrm{i}$	**M1**
		Obtain $\dfrac{7-\mathrm{i}}{5}$ or equivalent	**A1**
		Find modulus of w and hence of w^2	**M1**
		Confirm modulus is 2	**A1**
		Find argument of w and hence of w^2	**M1**
		Obtain –0.284 radians or $-16.3°$	**A1**
	Or 2	Square both sides to obtain $(-8+6\mathrm{i})w^2=-12+16\mathrm{i}$	**B1**
		Find w^2 using relevant conjugate	**M1**
		Use correct process for finding modulus of w^2	**M1**
		Confirm modulus is 2	**A1**
		Use correct process for finding argument of w^2	**M1**
		Obtain –0.284 radians or $-16.3°$	**A1**

	Answer	Mark	
Or 3	Find modulus of LHS and RHS	**M1**	
	Find argument of LHS and RHS	**M1**	
	Obtain $\sqrt{10}\,\mathrm{e}^{1.249\mathrm{i}}\ w = \sqrt{20}\,\mathrm{e}^{1.107\mathrm{i}}$ or equivalent	**A1**	
	Obtain $w = \sqrt{2}\,\mathrm{e}^{-0.1419\mathrm{i}}$ or equivalent	**A1**	
	Use correct process for finding w^2	**M1**	
	Obtain 2 and –0.284 radians or $-16.3°$	**A1**	
Or 4	Find moduli of $2+4\mathrm{i}$ and $1+3\mathrm{i}$	**M1**	
	Obtain $\sqrt{20}$ and $\sqrt{10}$	**A1**	
	Obtain $\lvert w^2 \rvert = 2$ correctly	**A1**	
	Find $\arg(2+4\mathrm{i})$ and $\arg(1+3\mathrm{i})$	**M1**	
	Use correct process for $\arg(w^2)$	**A1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 5	Let $w = a + \mathrm{i}b$, form and solve simultaneous equations in a and b	**M1**	
	$a = \frac{7}{5}$ and $b = -\frac{1}{5}$	**A1**	
	Find modulus of w and hence of w^2	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of w and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	
Or 6	Find w using conjugate of $1+3\mathrm{i}$	**M1**	
	Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
	Use $\lvert w^2 \rvert = w\overline{w}$	**M1**	
	Confirm modulus is 2	**A1**	
	Find argument of w and hence of w^2	**M1**	
	Obtain –0.284 radians or $-16.3°$	**A1**	[6]
(b)	Draw circle with centre the origin and radius 5	**B1**	
	Draw straight line parallel to imaginary axis in correct position	**B1**	
	Use relevant trigonometry on a correct diagram to find argument(s)	**M1**	
	Obtain $5\mathrm{e}^{\pm\frac{1}{3}\pi\mathrm{i}}$ or equivalents in required form	**A1**	[4]